

# Summarizing Data

DATA 606 - Statistics & Probability for Data Analytics

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# Announcements

- Mirian Lima talking about how to provide your judgement to agents at [New York Open Statistical Programming Meetup](#) group on September 28th at 7:00pm at NYU.
- We will hold our meetup at 5:00pm that night so anyone who wants to go can still go. The recording, as usual, will be uploaded right after the meetup ends.

# One Minute Paper Results

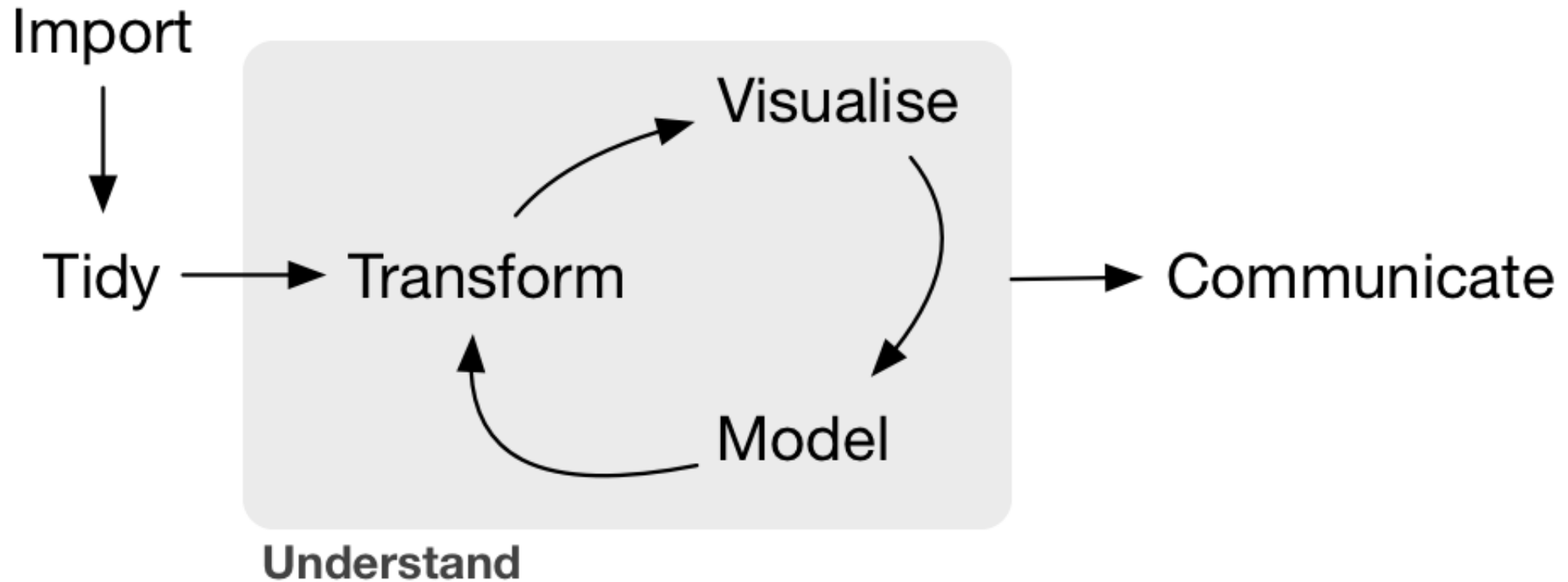
What was the most important thing you learned during this class?



## What important question remains unanswered for you?



# Workflow



Source: Wickham & Grolemund, 2017

# Tidy Data

“**TIDY DATA** is a standard way of mapping the meaning of a dataset to its structure.”

—HADLEY WICKHAM

## In tidy data:

- each variable forms a column
- each observation forms a row
- each cell is a single measurement

each column a variable

| id | name   | color  |
|----|--------|--------|
| 1  | floof  | gray   |
| 2  | max    | black  |
| 3  | cat    | orange |
| 4  | donut  | gray   |
| 5  | merlin | black  |
| 6  | panda  | calico |

each row an observation

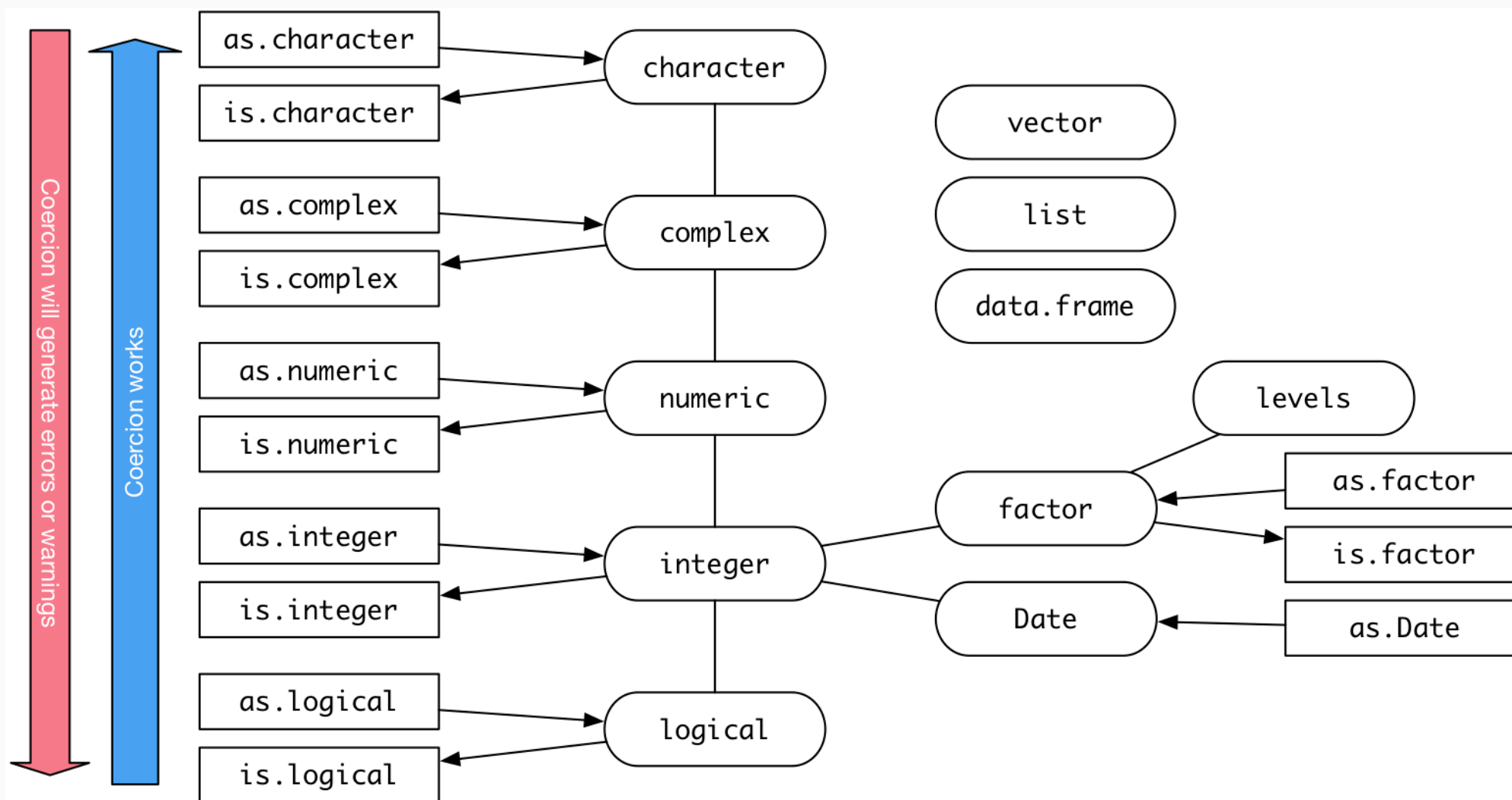
Wickham, H. (2014). Tidy Data. Journal of Statistical Software 59 (10). DOI: 10.18637/jss.v059.i10

# Types of Data

- Numerical (quantitative)
  - Continuous
  - Discrete
- Categorical (qualitative)
  - Regular categorical
  - Ordinal



# Data Types in R



# Data Types / Descriptives / Visualizations

| Data Type                  | Descriptive Stats                             | Visualization                |
|----------------------------|-----------------------------------------------|------------------------------|
| Continuous                 | mean, median, mode, standard deviation, IQR   | histogram, density, box plot |
| Discrete                   | contingency table, proportional table, median | bar plot                     |
| Categorical                | contingency table, proportional table         | bar plot                     |
| Ordinal                    | contingency table, proportional table, median | bar plot                     |
| Two quantitative           | correlation                                   | scatter plot                 |
| Two qualitative            | contingency table, chi-squared                | mosaic plot, bar plot        |
| Quantitative & Qualitative | grouped summaries, ANOVA, t-test              | box plot                     |

# Statistics

When describing a quantitative variable we are often interested in two things:

1. A measure of center
2. A measure of spread

The most common measures we will use in this class is the mean and median.

$$\bar{x} = \frac{\Sigma(x_i)}{n}$$

$$S^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

Note that in the numerator for the variance calculation we square the differences (also known as deviations). Squaring terms is common practice in statistics that serves two purposes:

1. It makes all the values positive.
2. It weighs observations that are further from the center more.

# Variance

Population Variance:

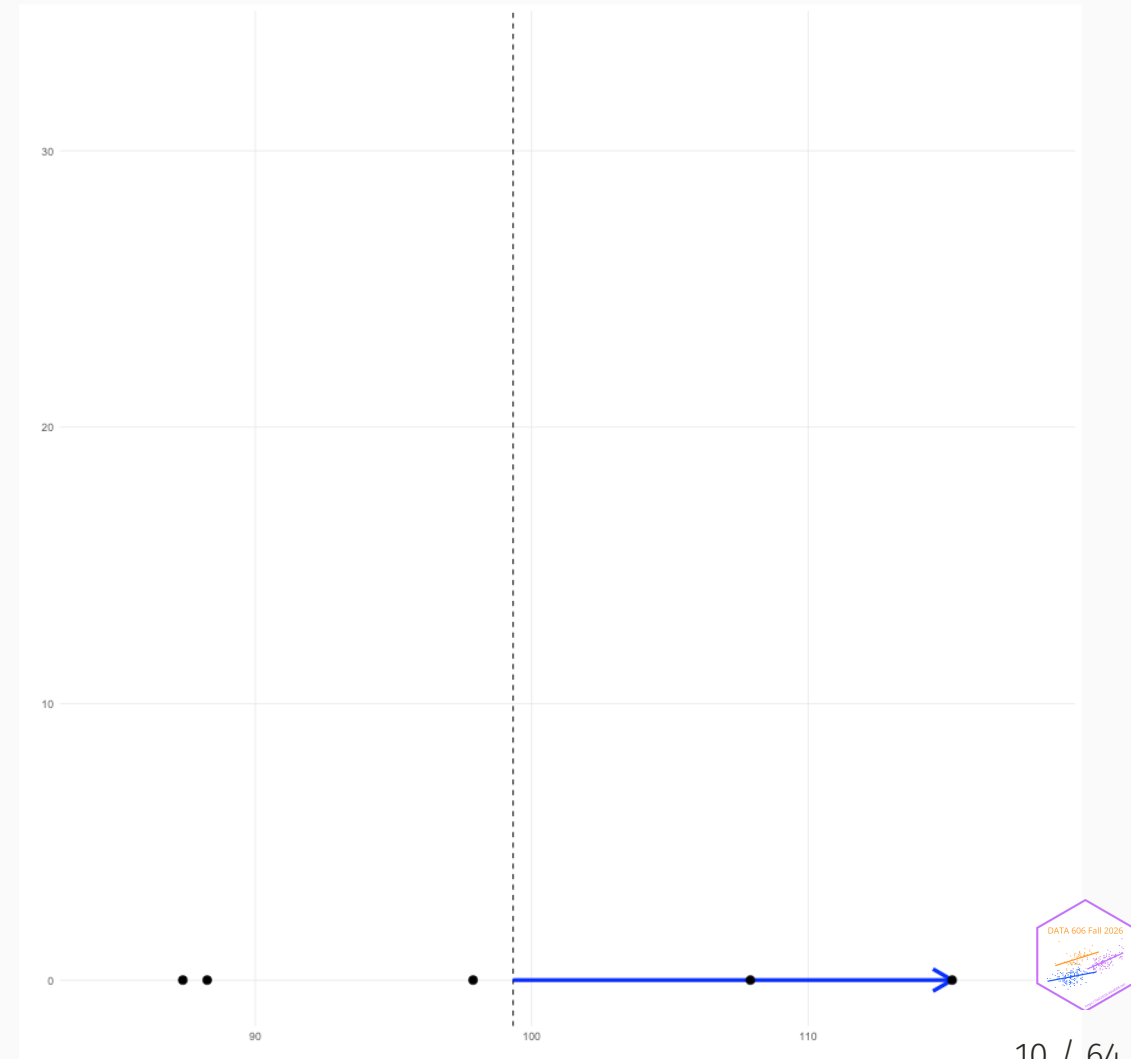
$$S^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$

Consider a dataset with five values (black points in the figure). For the largest value, the deviance is represented by the blue line (  $x_i - \bar{x}$  ).

See also:

<https://shiny.rit.albany.edu/stat/visualizess/>

<https://github.com/jbryer/VisualStats/>

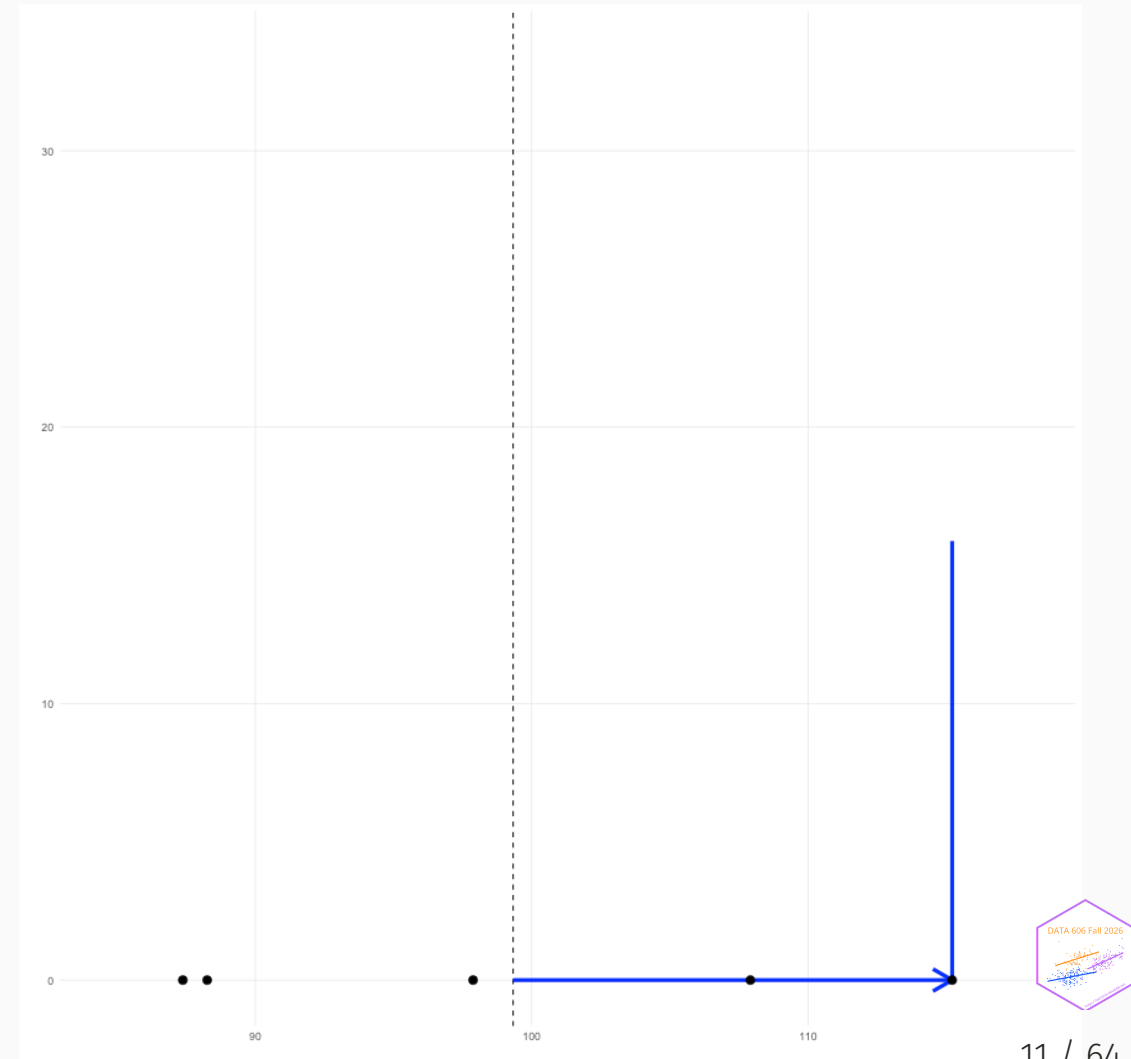


# Variance (cont.)

Population Variance:

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$

In the numerator, we square each of these deviances. We can conceptualize this as a square. Here, we add the deviance in the y direction.

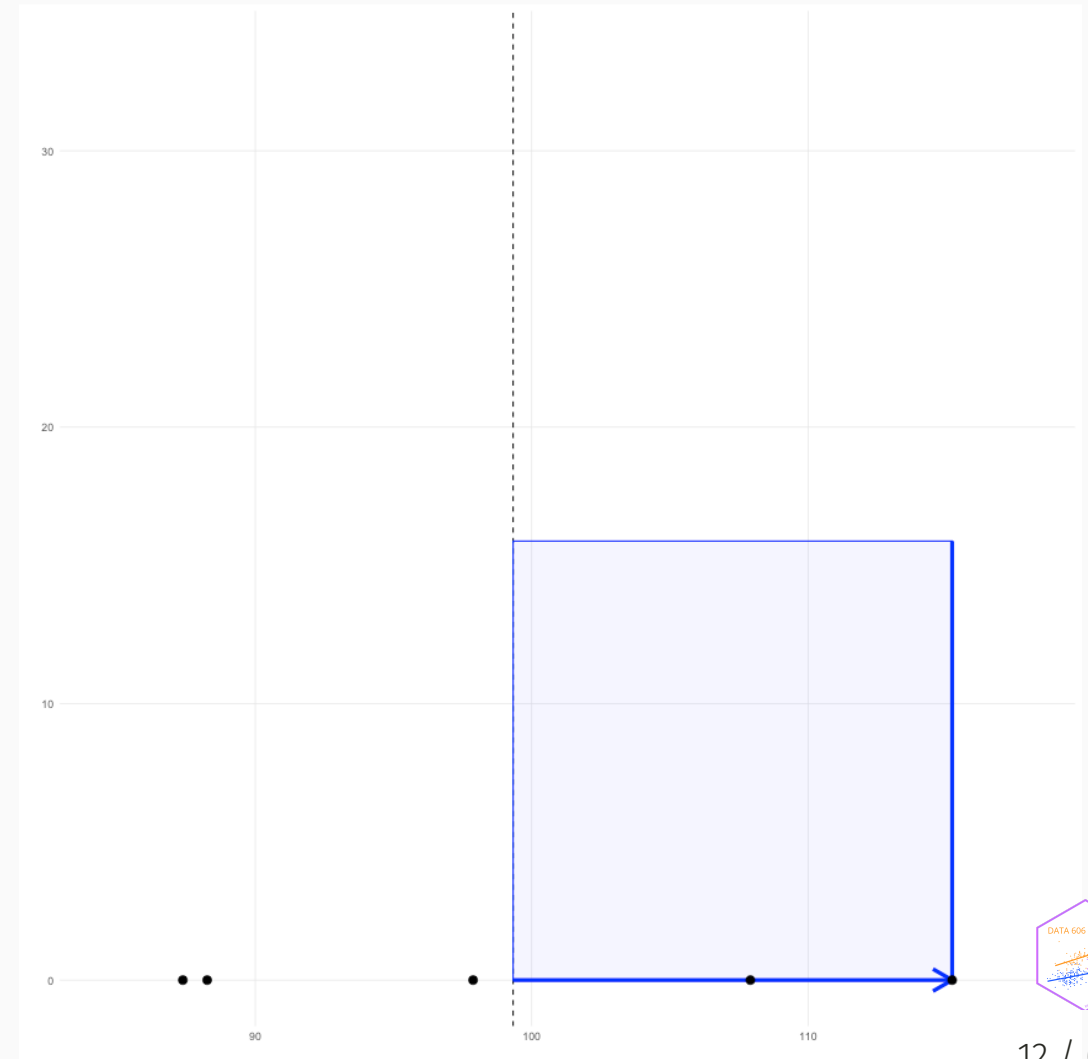


# Variance (cont.)

Population Variance:

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$

We end up with a square.

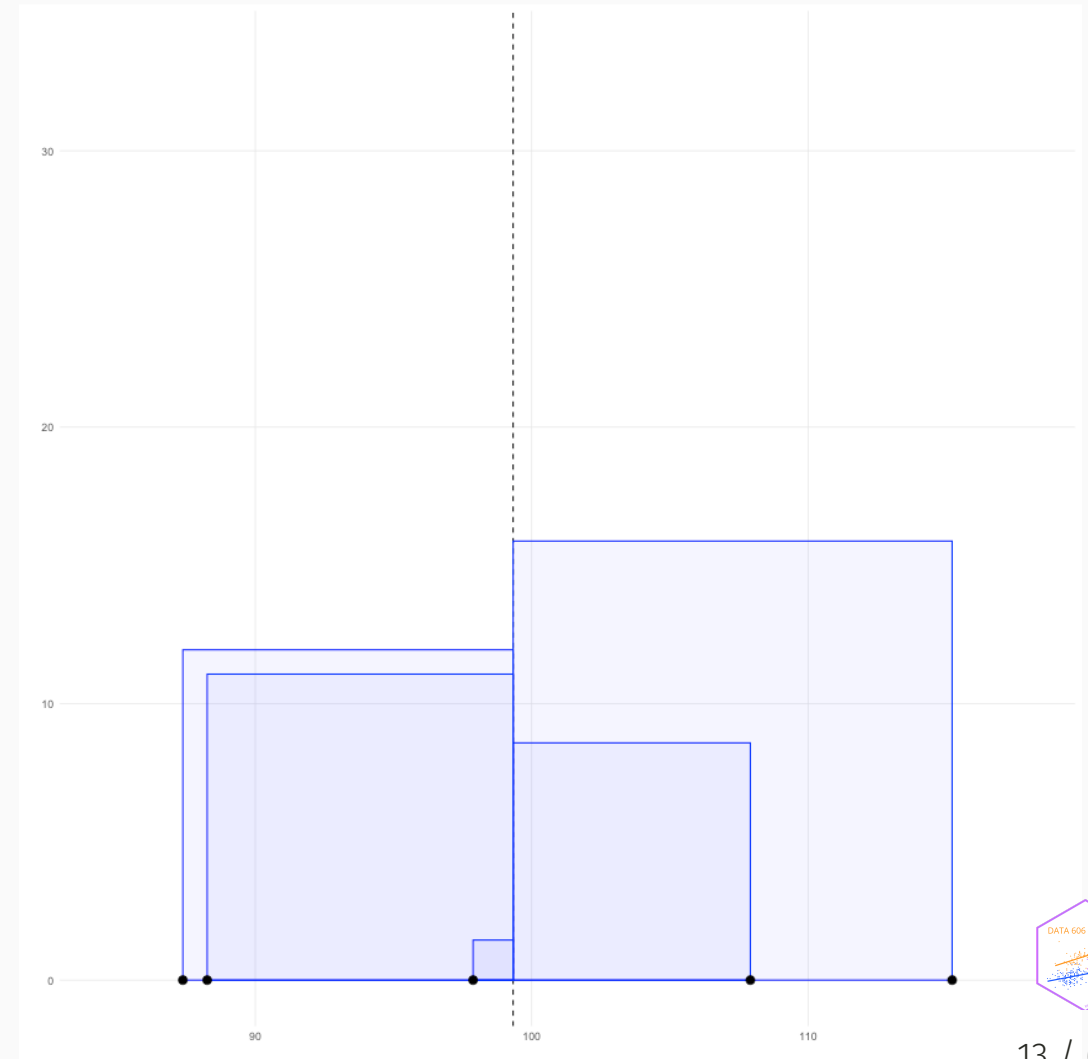


# Variance (cont.)

Population Variance:

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$

We can plot the squared deviance for all the data points. That is, each component in the numerator is the area of each of these squares.

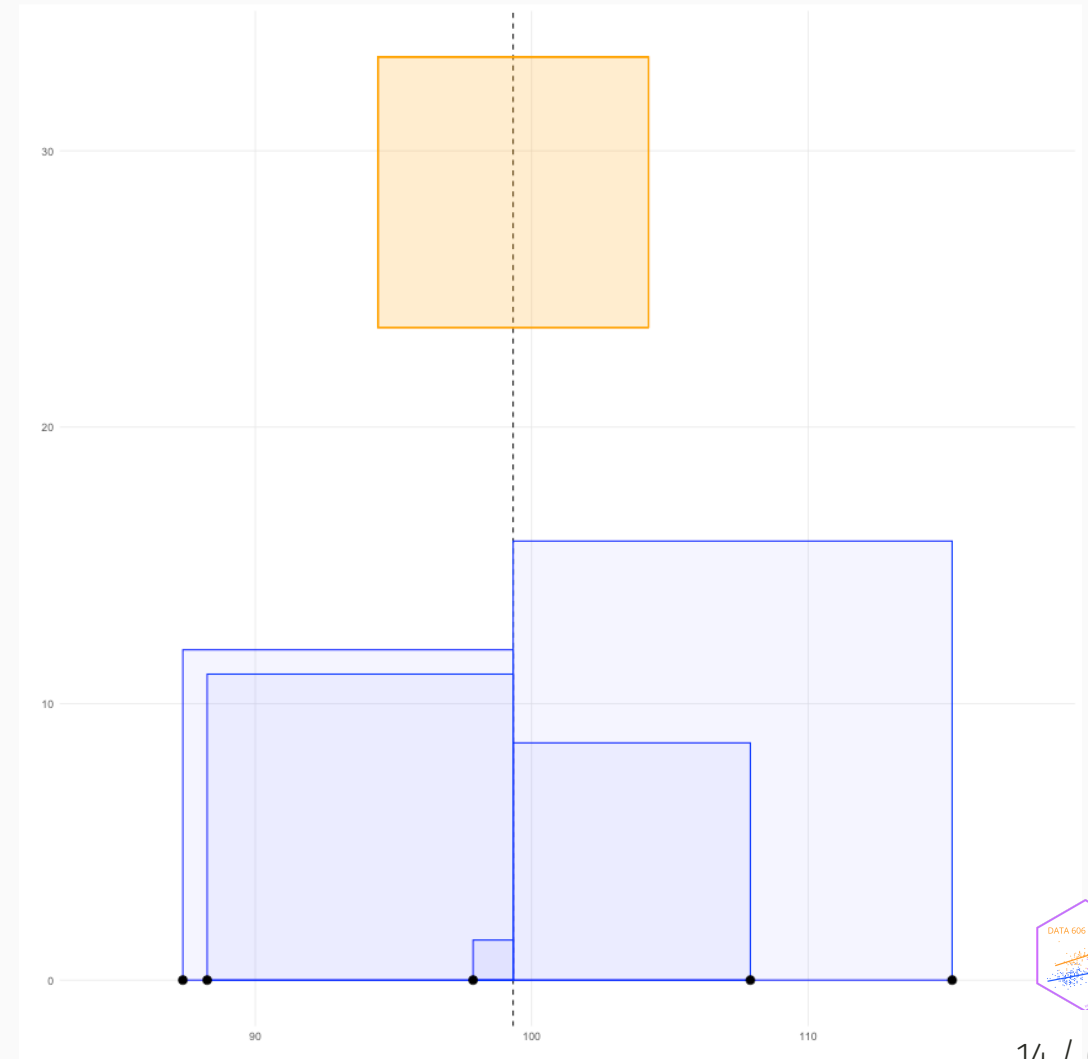


# Variance (cont.)

Population Variance:

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$

The variance is therefore the average of the area of all these squares, here represented by the orange square.



# Population versus Sample Variance

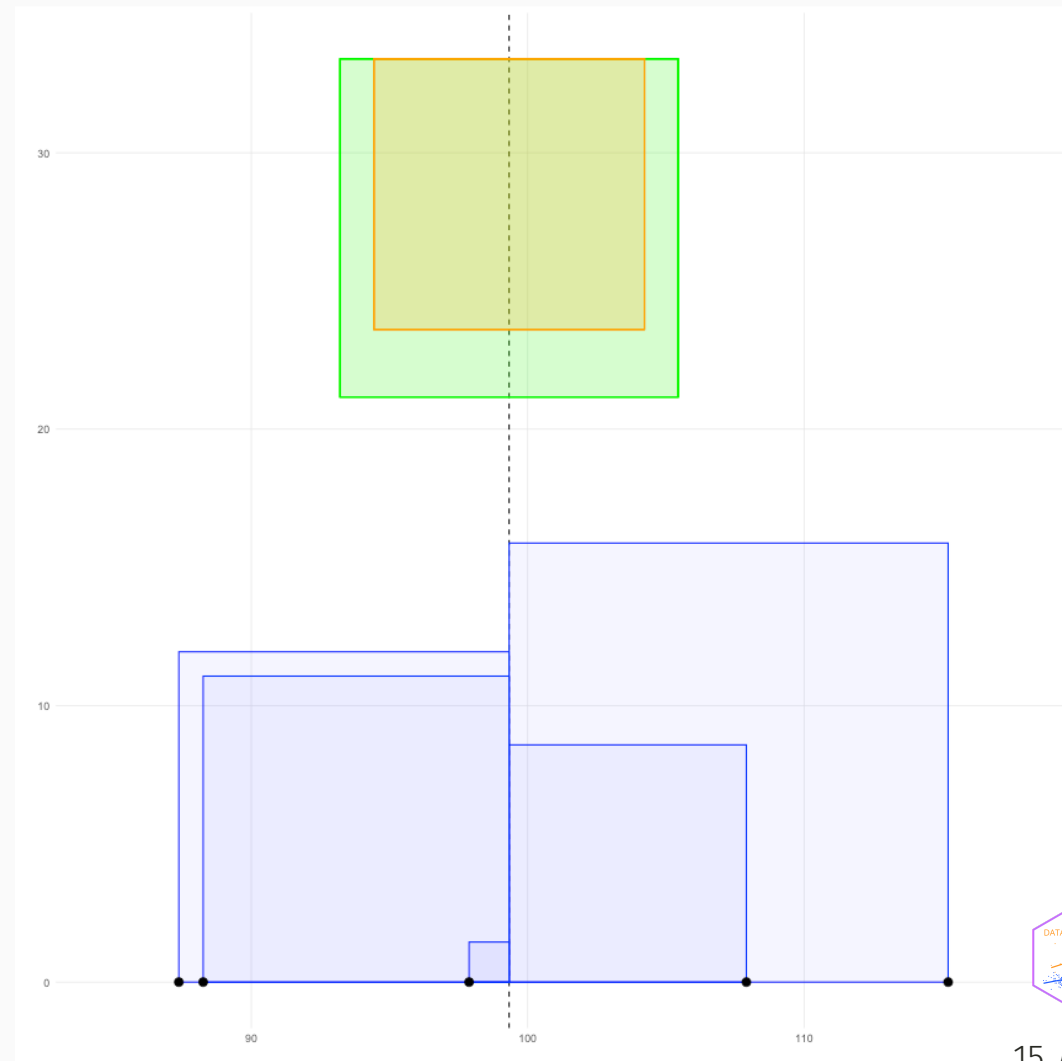
Typically we want the sample variance. The difference is we divide by  $n - 1$  to calculate the sample variance. This results in a slightly larger area (variance) then if we divide by  $n$ .

Population Variance (yellow):

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$

Sample Variance (green):

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$$



# Robust Statistics

Consider the following data randomly selected from the normal distribution:

```
set.seed(41)
x <- rnorm(30, mean = 100, sd = 15)
mean(x); sd(x)
```

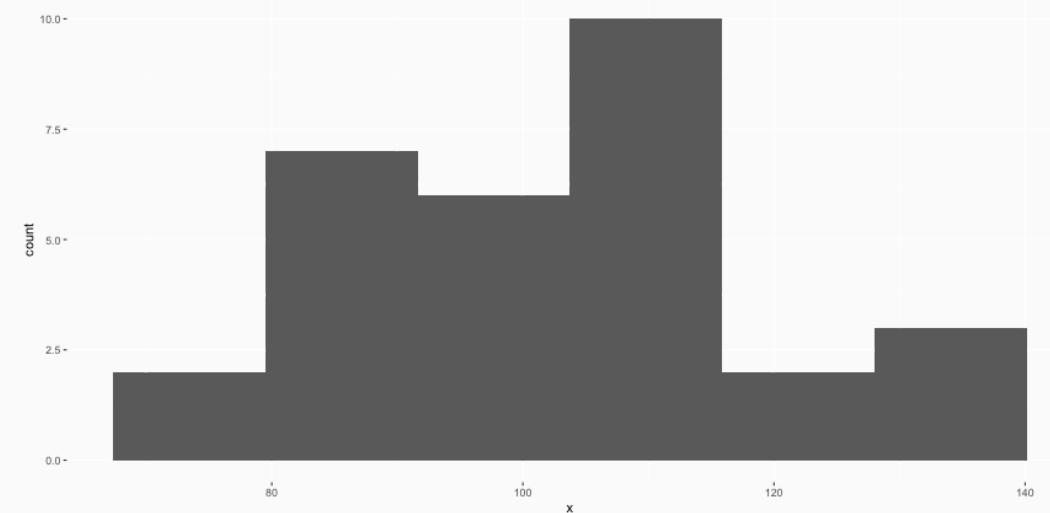
```
## [1] 103.1934
```

```
## [1] 16.8945
```

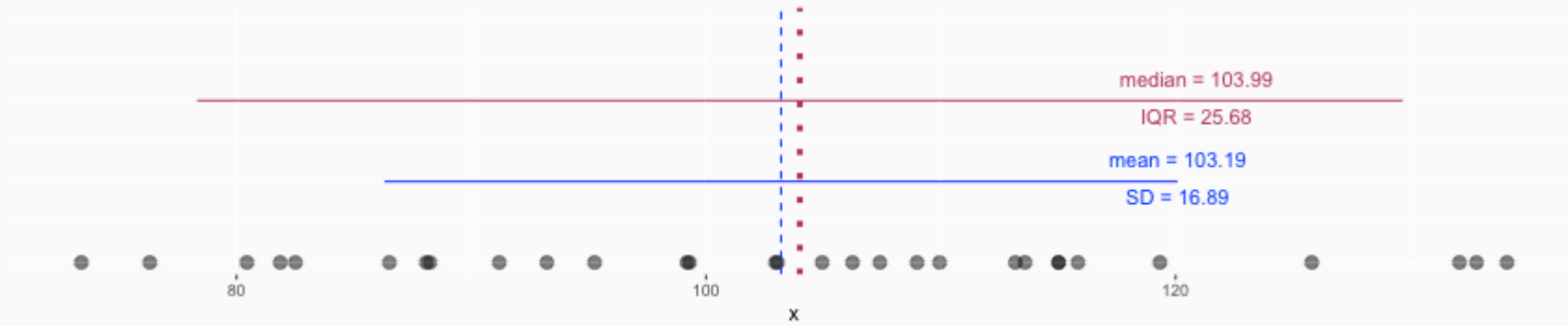
```
median(x); IQR(x)
```

```
## [1] 103.9947
```

```
## [1] 25.68004
```



# Robust Statistics



# Robust Statistics



Let's add an extreme value:

```
x <- c(x, 1000)
```



# Robust Statistics

Median and IQR are more robust to skewness and outliers than mean and SD. Therefore,

- for skewed distributions it is often more helpful to use median and IQR to describe the center and spread
- for symmetric distributions it is often more helpful to use the mean and SD to describe the center and spread

# About legosets



To install the brickset package:

```
remotes::install_github('jbryer/brickset')
```

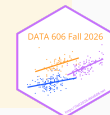
To load the load the legosets dataset.

```
data('legosets', package = 'brickset')
```

The legosets data has 21546 observations of 36 variables.

```
names(legosets)
```

```
## [1] "setID"          "number"          "numberVariant"
## [4] "name"           "year"            "theme"
## [7] "themeGroup"     "subtheme"        "category"
## [10] "released"       "pieces"          "minifigs"
## [13] "bricksetURL"    "rating"          "reviewCount"
## [16] "packagingType"  "availability"     "agerange_min"
## [19] "thumbnailURL"   "imageURL"        "US_retailPrice"
## [22] "US_dateFirstAvailable" "US_dateLastAvailable" "UK_retailPrice"
## [25] "UK_dateFirstAvailable" "UK_dateLastAvailable" "CA_retailPrice"
## [28] "CA_dateFirstAvailable" "CA_dateLastAvailable" "DE_retailPrice"
## [31] "DE_dateFirstAvailable" "DE_dateLastAvailable" "height"
## [34] "width"          "depth"           "weight"
```



# Structure (str)



```
str(legosets)
```

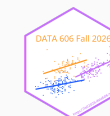
```
## 'data.frame':    21546 obs. of  36 variables:
## $ setID          : int  7693 7695 7697 7698 25534 7418 7419 6020 22704 7421 ...
## $ number         : chr  "1" "2" "3" "4" ...
## $ numberVariant   : int  8 8 6 4 6 1 1 1 3 4 ...
## $ name           : chr  "Small house set" "Medium house set" "Large house set" ...
## $ year           : int  1970 1970 1970 1970 1970 1970 1970 1970 1970 ...
## $ theme          : chr  "Minitalia" "Minitalia" "Minitalia" "Minitalia" ...
## $ themeGroup      : chr  "Vintage" "Vintage" "Vintage" "Vintage" ...
## $ subtheme       : chr  NA NA NA NA ...
## $ category        : chr  "Normal" "Normal" "Normal" "Normal" ...
## $ released        : logi  TRUE TRUE TRUE TRUE TRUE TRUE ...
## $ pieces          : int  67 109 158 233 NA 1 1 60 65 NA ...
## $ minifigs        : int  NA NA NA NA NA NA NA NA NA NA ...
## $ bricksetURL      : chr  "https://brickset.com/sets/1-8" "https://brickset.com/sets/2-8" "https://brickset.com/sets/3-6" "https://brickset.com/sets/4-4" ...
## $ rating          : num  0 0 0 0 0 0 0 0 0 ...
## $ reviewCount      : int  0 0 1 0 0 0 0 0 0 ...
## $ packagingType    : chr  "{Not specified}" "{Not specified}" "{Not specified}" "{Not specified}" ...
## $ availability      : chr  "{Not specified}" "{Not specified}" "{Not specified}" "{Not specified}" ...
## $ agerange_min     : int  NA NA NA NA NA NA NA NA NA NA ...
## $ thumbnailURL     : chr  "https://images.brickset.com/sets/small/1-8.jpg" "https://images.brickset.com/sets/small/2-8.jpg" "https://images.brickset.com/sets/small/3-6.jpg" "https://images.brickset.com/sets/small/4-4.jpg" ...
## $ imageURL         : chr  "https://images.brickset.com/sets/images/1-8.jpg" "https://images.brickset.com/sets/images/2-8.jpg" "https://images.brickset.com/sets/images/3-6.jpg" "https://images.brickset.com/sets/images/4-4.jpg" ...
## $ US_retailPrice    : num  NA NA NA NA NA NA NA NA NA NA ...
## $ US_dateFirstAvailable: Date, format: NA NA ...
## $ US_dateLastAvailable : Date, format: NA NA ...
## $ UK_retailPrice    : num  NA NA NA NA NA NA NA NA NA NA ...
## $ UK_dateFirstAvailable: Date, format: NA NA ...
## $ UK_dateLastAvailable : Date, format: NA NA ...
## $ CA_retailPrice    : num  NA NA NA NA NA NA NA NA NA NA ...
## $ CA_dateFirstAvailable: Date, format: NA NA ...
## $ CA_dateLastAvailable : Date, format: NA NA ...
## $ DE_retailPrice    : num  NA NA NA NA NA NA NA NA NA NA ...
## $ DE_dateFirstAvailable: Date, format: NA NA ...
## $ DE_dateLastAvailable : Date, format: NA NA ...
## $ height           : num  NA NA NA NA NA ...
## $ width            : num  NA NA NA NA NA ...
```



# RStudio Environment tab can help



| Environment              | History                    | Connections                                                    | Git | Tutorial |
|--------------------------|----------------------------|----------------------------------------------------------------|-----|----------|
| R Global Environment     |                            |                                                                |     |          |
| Data                     |                            |                                                                |     |          |
| legosets                 | 16355 obs. of 34 variables |                                                                |     |          |
| \$ setID                 | : int                      | 7693 7695 7697 7698 25534 7418 7419 6020 22704 7421 ...        |     |          |
| \$ name                  | : chr                      | "Small house set" "Medium house set" "Medium house set" "L...  |     |          |
| \$ year                  | : int                      | 1970 1970 1970 1970 1970 1970 1970 1970 1970 1970 ...          |     |          |
| \$ theme                 | : chr                      | "Minitalia" "Minitalia" "Minitalia" "Minitalia" ...            |     |          |
| \$ themeGroup            | : chr                      | "Vintage" "Vintage" "Vintage" "Vintage" ...                    |     |          |
| \$ subtheme              | : chr                      | NA NA NA NA ...                                                |     |          |
| \$ category              | : chr                      | "Normal" "Normal" "Normal" "Normal" ...                        |     |          |
| \$ released              | : logi                     | TRUE TRUE TRUE TRUE TRUE TRUE ...                              |     |          |
| \$ pieces                | : int                      | 67 109 158 233 NA 1 1 60 65 NA ...                             |     |          |
| \$ minifigs              | : int                      | NA NA NA NA NA NA NA NA ...                                    |     |          |
| \$ bricksetURL           | : chr                      | "https://brickset.com/sets/1-8" "https://brickset.com/sets...  |     |          |
| \$ rating                | : num                      | 0 0 0 0 0 0 0 0 0 ...                                          |     |          |
| \$ reviewCount           | : int                      | 0 0 1 0 0 0 0 1 0 ...                                          |     |          |
| \$ packagingType         | : chr                      | "{Not specified}" "{Not specified}" "{Not specified}" "{No...  |     |          |
| \$ availability          | : chr                      | "{Not specified}" "{Not specified}" "{Not specified}" "{No...  |     |          |
| \$ ageRange_min          | : int                      | NA NA NA NA NA NA NA NA NA ...                                 |     |          |
| \$ US_retailPrice        | : num                      | NA NA NA NA NA 1.99 NA NA 4.99 NA ...                          |     |          |
| \$ US_dateFirstAvailable | : Date, format:            | NA NA NA NA ...                                                |     |          |
| \$ US_dateLastAvailable  | : Date, format:            | NA NA NA NA ...                                                |     |          |
| \$ UK_retailPrice        | : num                      | NA NA NA NA NA NA NA NA NA ...                                 |     |          |
| \$ UK_dateFirstAvailable | : Date, format:            | NA NA NA NA ...                                                |     |          |
| \$ UK_dateLastAvailable  | : Date, format:            | NA NA NA NA ...                                                |     |          |
| \$ CA_retailPrice        | : num                      | NA NA NA NA NA NA NA NA NA ...                                 |     |          |
| \$ CA_dateFirstAvailable | : Date, format:            | NA NA NA NA ...                                                |     |          |
| \$ CA_dateLastAvailable  | : Date, format:            | NA NA NA NA ...                                                |     |          |
| \$ DE_retailPrice        | : num                      | NA NA NA NA NA NA NA NA NA ...                                 |     |          |
| \$ DE_dateFirstAvailable | : Date, format:            | NA NA NA NA ...                                                |     |          |
| \$ DE_dateLastAvailable  | : Date, format:            | NA NA NA NA ...                                                |     |          |
| \$ height                | : num                      | NA NA NA NA NA ...                                             |     |          |
| \$ width                 | : num                      | NA NA NA NA NA ...                                             |     |          |
| \$ depth                 | : num                      | NA NA NA NA NA NA NA 5.08 NA ...                               |     |          |
| \$ weight                | : num                      | NA NA NA NA NA NA NA NA NA ...                                 |     |          |
| \$ thumbnailURL          | : chr                      | "https://images.brickset.com/sets/small/1-8.jpg" "https://...  |     |          |
| \$ imageURL              | : chr                      | "https://images.brickset.com/sets/images/1-8.jpg" "https://... |     |          |



# Table View

Show 

10

 entries

Search:

|    | setID | name                                                    | year | theme                   | themeGroup       | category | US_retailPrice | pieces | minifigs | rating |
|----|-------|---------------------------------------------------------|------|-------------------------|------------------|----------|----------------|--------|----------|--------|
| 1  | 9788  | Moose                                                   | 2012 | Promotional             | Miscellaneous    | Normal   |                | 27     |          | 0      |
| 2  | 28901 | LEGO Minifigures - The LEGO Movie 2 Series {Random bag} | 2019 | Collectable Minifigures | Miscellaneous    | Random   |                |        |          | 3.8    |
| 3  | 24155 | Temple of Airjitzu                                      | 2015 | Ninjago                 | Action/Adventure | Normal   | 199.99         | 2028   | 13       | 4.6    |
| 4  | 1324  | Marie in Rainbow Skirt                                  | 1998 | Scala                   | Girls            | Normal   |                | 10     | 1        | 0      |
| 5  | 4184  | Fire Engine                                             | 1995 | Technic                 | Technical        | Normal   |                | 424    | 2        | 3.6    |
| 6  | 28548 | Kai Minifigure Alarm Clock                              | 2018 | Gear                    | Miscellaneous    | Gear     |                |        |          | 0      |
| 7  | 31973 | LEGO Meet the Minifigures                               | 2022 | Books                   | Miscellaneous    | Book     |                | 8      |          | 0      |
| 8  | 4118  | Universal Set with Flex System                          | 1991 | Technic                 | Technical        | Normal   |                | 313    |          | 3.7    |
| 9  | 51241 | Mario Kart - Mario & Standard Kart                      | 2025 | Super Mario             | Licensed         | Normal   | 169.99         | 1972   |          | 4.4    |
| 10 | 31879 | Luigi Key Chain                                         | 2021 | Gear                    | Miscellaneous    | Gear     |                |        |          | 0      |

Showing 1 to 10 of 100 entries

Previous

1

2345...10Next

# Data Wrangling Cheat Sheet



## Data Transformation with dplyr : : CHEAT SHEET



**dplyr** functions work with pipes and expect **tidy data**. In tidy data:



Each **variable** is in its own **column**



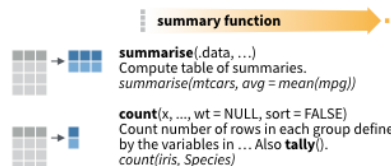
Each **observation**, or **case**, is in its own **row**



**x %>% f(y)** becomes **f(x, y)**

### Summarise Cases

These apply **summary functions** to columns to create a new table of summary statistics. Summary functions take vectors as input and return one value (see back).



#### VARIATIONS

**summarise\_all()** - Apply funs to every column.  
**summarise\_at()** - Apply funs to specific columns.  
**summarise\_if()** - Apply funs to all cols of one type.

### Group Cases

Use **group\_by()** to create a "grouped" copy of a table. dplyr functions will manipulate each "group" separately and then combine the results.



**group\_by(data, ..., add = FALSE)**  
Returns copy of table grouped by ...  
`g_iris <- group_by(iris, Species)`

**ungroup(x, ...)**  
Returns ungrouped copy of table.  
`ungroup(g_iris)`

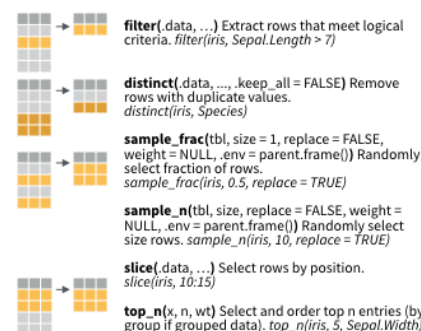


RStudio® is a trademark of RStudio, Inc. • CC BY SA RStudio • info@rstudio.com • 844-448-1212 • rstudio.com • Learn more with `browseVignettes(package = c("dplyr", "tibble"))` • dplyr 0.7.0 • tibble 1.2.0 • Updated: 2017-03

### Manipulate Cases

#### EXTRACT CASES

Row functions return a subset of rows as a new table.



#### Logical and boolean operators to use with filter()

|   |    |          |      |   |       |
|---|----|----------|------|---|-------|
| < | <= | is.na()  | %in% |   | xor() |
| > | >= | !is.na() | !    | & |       |

See **?base:logic** and **?Comparison** for help.

#### ARRANGE CASES

**arrange(data, ...)** Order rows by values of a column or columns (low to high), use with **desc()** to order from high to low.  
`arrange(mtcars, mpg)`  
`arrange(mtcars, desc(mpg))`

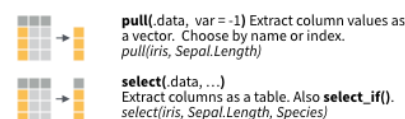
#### ADD CASES

**add\_row(data, ..., before = NULL, after = NULL)**  
Add one or more rows to a table.  
`add_row(faithful, eruptions = 1, waiting = 1)`

### Manipulate Variables

#### EXTRACT VARIABLES

Column functions return a set of columns as a new vector or table.

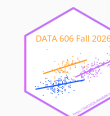
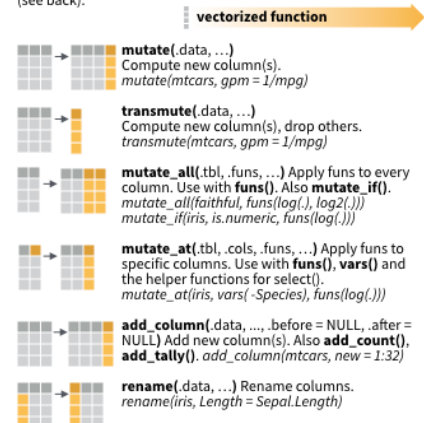


Use these helpers with **select()**, e.g. `select(iris, starts_with("Sepal"))`

|                         |                                 |                               |
|-------------------------|---------------------------------|-------------------------------|
| <b>contains(match)</b>  | <b>num_range(prefix, range)</b> | z, e.g. <code>mpg:cyl</code>  |
| <b>ends_with(match)</b> | <b>one_of(...)</b>              | z, e.g. <code>-Species</code> |
| <b>matches(match)</b>   | <b>starts_with(match)</b>       |                               |

#### MAKE NEW VARIABLES

These apply **vectorized functions** to columns. Vectorized funs take vectors as input and return vectors of the same length as output (see back).



## R Syntax Comparison : : CHEAT SHEET

### Dollar sign syntax

`goal(data$x, data$y)`

#### SUMMARY STATISTICS:

one continuous variable:  
`mean(mtcars$mpg)`

one categorical variable:  
`table(mtcars$cyl)`

two categorical variables:  
`table(mtcars$cyl, mtcars$am)`

one continuous, one categorical:  
`mean(mtcars$mpg[mtcars$cyl==4])`  
`mean(mtcars$mpg[mtcars$cyl==6])`  
`mean(mtcars$mpg[mtcars$cyl==8])`

#### PLOTTING:

one continuous variable:  
`hist(mtcars$disp)`

`boxplot(mtcars$disp)`

one categorical variable:  
`barplot(table(mtcars$cyl))`

two continuous variables:  
`plot(mtcars$disp, mtcars$mpg)`

two categorical variables:  
`mosaicplot(table(mtcars$am, mtcars$cyl))`

one continuous, one categorical:  
`histogram(mtcars$disp[mtcars$cyl==4])`  
`histogram(mtcars$disp[mtcars$cyl==6])`  
`histogram(mtcars$disp[mtcars$cyl==8])`

`boxplot(mtcars$disp[mtcars$cyl==4])`  
`boxplot(mtcars$disp[mtcars$cyl==6])`  
`boxplot(mtcars$disp[mtcars$cyl==8])`

#### WRANGLING:

subsetting:  
`mtcars[mtcars$mpg>30, ]`

making a new variable:  
`mtcars$efficient[mtcars$mpg>30] <- TRUE`  
`mtcars$efficient[mtcars$mpg<30] <- FALSE`

### Formula syntax

`goal(y~x|z, data=data, group=w)`

#### SUMMARY STATISTICS:

one continuous variable:  
`mosaic::mean(~mpg, data=mtcars)`

one categorical variable:  
`mosaic::tally(~cyl, data=mtcars)`

two categorical variables:  
`mosaic::tally(cyl~am, data=mtcars)`

one continuous, one categorical:  
`mosaic::mean(mpg~cyl, data=mtcars)`

tilde

#### PLOTTING:

one continuous variable:  
`lattice::histogram(~disp, data=mtcars)`

`lattice::bwplot(~disp, data=mtcars)`

one categorical variable:  
`mosaic::bargraph(~cyl, data=mtcars)`

two continuous variables:  
`lattice::xyplot(mpg~disp, data=mtcars)`

two categorical variables:  
`mosaic::bargraph(~am, data=mtcars, group=cyl)`

one continuous, one categorical:  
`lattice::histogram(~disp|cyl, data=mtcars)`  
`lattice::bwplot(cyl~disp, data=mtcars)`

The variety of R syntaxes give you many ways to “say” the same thing

read across the cheatsheet to see how different syntaxes approach the same problem

### Tidyverse syntax

`data %>% goal(x)`

#### SUMMARY STATISTICS:

one continuous variable:  
`mtcars %>% dplyr::summarize(mean(mpg))`

one categorical variable:  
`mtcars %>% dplyr::group_by(cyl) %>% dplyr::summarize(n())`

the pipe

two categorical variables:  
`mtcars %>% dplyr::group_by(cyl, am) %>% dplyr::summarize(n())`

one continuous, one categorical:  
`mtcars %>% dplyr::group_by(cyl) %>% dplyr::summarize(mean(mpg))`

#### PLOTTING:

one continuous variable:  
`ggplot2::qplot(x=mpg, data=mtcars, geom = "histogram")`

`ggplot2::qplot(y=disp, x=1, data=mtcars, geom="boxplot")`

one categorical variable:  
`ggplot2::qplot(x=cyl, data=mtcars, geom="bar")`

two continuous variables:  
`ggplot2::qplot(x=disp, y=mpg, data=mtcars, geom="point")`

two categorical variables:  
`ggplot2::qplot(x=factor(cyl), data=mtcars, geom="bar") + facet_grid(.~am)`

one continuous, one categorical:  
`ggplot2::qplot(x=disp, data=mtcars, geom = "histogram") + facet_grid(.~cyl)`

`ggplot2::qplot(y=disp, x=factor(cyl), data=mtcars, geom="boxplot")`

#### WRANGLING:

subsetting:  
`mtcars %>% dplyr::filter(mpg>30)`

making a new variable:  
`mtcars <- mtcars %>% dplyr::mutate(efficient = if_else(mpg>30, TRUE, FALSE))`

# Pipes %>% and |>



The pipe operator (`%>%`) introduced with the `magrittr` R package allows for the chaining of R operations. As of version 4.1, R now has a native pipe operator (`|>`). They take the output from the left-hand side and passes it as the first parameter to the function on the right-hand side.



You can do this in two steps:

```
tab_out <- table(legosets$category)
prop.table(tab_out)
```

Using the pipe (`|>`) operator we can chain these calls in a what is arguably a more readable format:

```
legosets$category |> table() |> prop.table()
```

Or as nested function calls.

```
prop.table(table(legosets$category))
```

```
##
##      Book  Collection   Extended      Gear      Normal      Other
## 0.035087719 0.029889539 0.036248027 0.158869396 0.668894458 0.067205050
##      Random
## 0.003805811
```

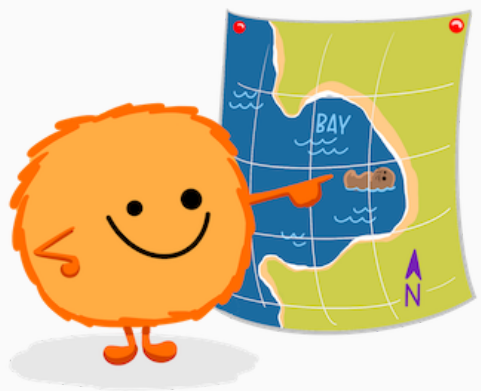


## dplyr::filter()

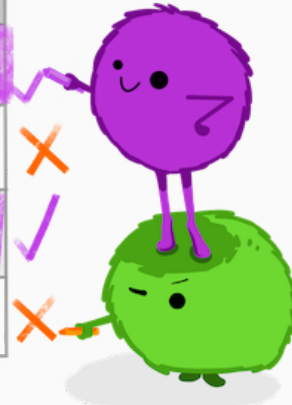
KEEP ROWS THAT  
satisfy  
your **CONDITIONS**

keep rows from... this data... ONLY IF... type MATCHES "otter" AND site MATCHES "bay"

```
filter(df, type == "otter" & site == "bay")
```



| type  | food    | site    |
|-------|---------|---------|
| otter | urchin  | bay     |
| shark | seal    | channel |
| otter | abalone | bay     |
| otter | crab    | wharf   |



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# Logical Operators

- `!a` - TRUE if `a` is FALSE
- `a == b` - TRUE if `a` and `b` are equal
- `a != b` - TRUE if `a` and `b` are not equal
- `a > b` - TRUE if `a` is larger than `b`, but not equal
- `a >= b` - TRUE if `a` is larger or equal to `b`
- `a < b` - TRUE if `a` is smaller than `b`, but not equal
- `a <= b` - TRUE if `a` is smaller or equal to `b`
- `a %in% b` - TRUE if `a` is in `b` where `b` is a vector

```
which( letters %in% c('a','e','i','o','u') )
```

```
## [1] 1 5 9 15 21
```

- `a | b` - TRUE if `a` or `b` are TRUE
- `a & b` - TRUE if `a` and `b` are TRUE
- `isTRUE(a)` - TRUE if `a` is TRUE

## dplyr

```
mylego <- legosets |> filter(themeGroup == 'Educational' & year > 2015)
```

## Base R

```
mylego <- legosets[legosets$themeGroup == 'Educational' & legosets$year > 2015,]
```

---

```
nrow(mylego)
```

```
## [1] 121
```

## dplyr

```
mylego <- mylego |> select(setID, pieces, theme, availability, US_retailPrice, minifigs)
```

## Base R

```
mylego <- mylego[,c('setID', 'pieces', 'theme', 'availability', 'US_retailPrice', 'minifigs')]
```

```
head(mylego, n = 4)
```

| ##   | setID | pieces | theme     | availability    | US_retailPrice | minifigs |
|------|-------|--------|-----------|-----------------|----------------|----------|
| ## 1 | 26803 | 109    | Education | {Not specified} | NA             | 6        |
| ## 2 | 26277 | 188    | Education | Educational     | 94.95          | NA       |
| ## 3 | 27742 | 160    | Education | {Not specified} | NA             | NA       |
| ## 4 | 26805 | 1000   | Education | {Not specified} | NA             | NA       |

# Relocate



dplyr<sup>1.0.0</sup>::relocate()  
move COLUMNS around!

Default: move to FRONT  
or move to  
.before or .after  
A SPECIFIED COLUMN!



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## dplyr

```
mylego |> relocate(where(is.numeric), .after = where(is.character)) |> head(n = 3)
```

```
##           theme      availability setID pieces US_retailPrice minifigs
## 1 Education {Not specified} 26803    109           NA           6
## 2 Education      Educational 26277    188          94.95          NA
## 3 Education {Not specified} 27742    160           NA           NA
```

## Base R

```
mylego2 <- mylego[,c('theme', 'availability', 'setID', 'pieces', 'US_retailPrice', 'minifigs')]
head(mylego2, n = 3)
```

```
##           theme      availability setID pieces US_retailPrice minifigs
## 1 Education {Not specified} 26803    109           NA           6
## 2 Education      Educational 26277    188          94.95          NA
## 3 Education {Not specified} 27742    160           NA           NA
```

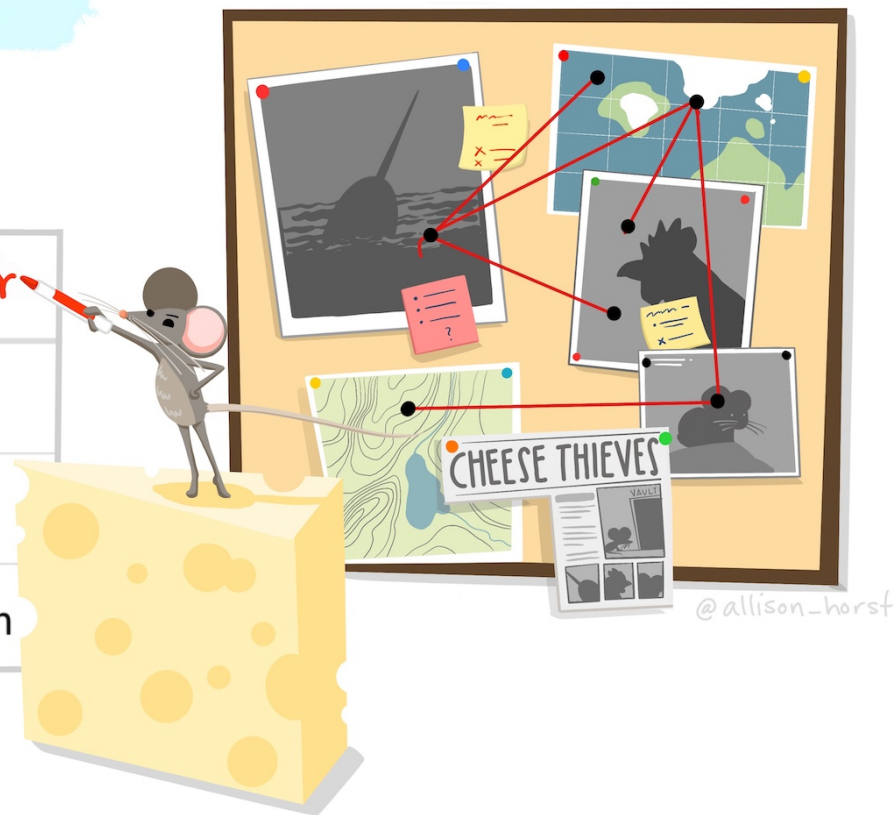
## dplyr::rename()

RENAME COLUMNS\*

```
df %>% rename(lair=site)
```

| <del>species</del> nemesis | status  | <del>site</del> lair |
|----------------------------|---------|----------------------|
| narwhal                    | unknown | ocean                |
| chicken                    | active  | coop                 |
| pika                       | active  | mountain             |

\*See `rename_with()` to rename using a function.



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# Rename



## dplyr

```
mylego |> dplyr::rename(USD = US_retailPrice) |> head(n = 3)
```

```
##   setID pieces   theme   availability   USD minifigs
## 1 26803   109 Education {Not specified}   NA         6
## 2 26277   188 Education   Educational 94.95        NA
## 3 27742   160 Education {Not specified}   NA        NA
```

## Base R

```
names(mylego2)[5] <- 'USD'
head(mylego2, n = 3)
```

```
##           theme   availability setID pieces   USD minifigs
## 1 Education {Not specified} 26803   109   NA         6
## 2 Education   Educational 26277   188 94.95        NA
## 3 Education {Not specified} 27742   160   NA        NA
```



# Mutate



## dplyr

```
mylego |> filter(!is.na(pieces) & !is.na(US_retailPrice)) |>
  mutate(Price_per_piece = US_retailPrice / pieces) |> head(n = 3)
```

```
##   setID pieces    theme availability US_retailPrice minifigs Price_per_piece
## 1 26277   188 Education Educational      94.95         NA      0.5050532
## 2 25949   280 Education Educational    224.95         NA      0.8033929
## 3 25954     1 Education Educational     14.95         NA     14.9500000
```

## Base R

```
mylego2 <- mylego[!is.na(mylego$US_retailPrice) & !is.na(mylego$Price_per_piece),]
mylego2$Price_per_piece <- mylego2$Price_per_piece / mylego2$US_retailPrice
head(mylego2, n = 3)
```

```
## [1] setID      pieces      theme      availability
## [5] US_retailPrice minifigs    Price_per_piece
## <0 rows> (or 0-length row.names)
```

# Group By and Summarize



```
legosets |> group_by(themeGroup) |> summarize(mean_price = mean(US_retailPrice, na.rm = TRUE),  
                                              sd_price = sd(US_retailPrice, na.rm = TRUE),  
                                              median_price = median(US_retailPrice, na.rm = TRUE),  
                                              n = n(),  
                                              missing = sum(is.na(US_retailPrice)))
```

```
## # A tibble: 17 × 6  
##   themeGroup      mean_price sd_price median_price      n missing  
##   <chr>          <dbl>    <dbl>         <dbl> <int>  <int>  
## 1 Action/Adventure  43.0    41.9         30.0  1620   845  
## 2 Art and crafts   41.0    53.0         20.0   104     9  
## 3 Basic           22.8    19.4         15.0   884   733  
## 4 Constraction    16.4    12.4         13.0   503   285  
## 5 Educational     185.    188.        138.   546   508  
## 6 Girls           35.8    24.0         23.0   240   227  
## 7 Historical       34.2    32.4         20.0   474   401  
## 8 Junior          22.0    10.1         20.0   228   165  
## 9 Licensed        55.5    72.9         35.0  3353  1245  
## 10 Miscellaneous   23.4    33.5         15.0  7151  4501  
## 11 Model making    86.2    99.4         50.0   919   422  
## 12 Modern day     39.8    36.8         30.0  2669  1594  
## 13 Pre-school     31.7    23.2         25.0  1613  1108  
## 14 Racing         26.4    26.8         15.0   266   176
```



# Describe and Describe By

```
library(psych)
describe(legosets$US_retailPrice)
```

```
##      vars      n mean      sd median trimmed      mad      min      max range skew kurtosis
## X1      1 8674 42.26 59.75  24.99   30.25 22.24  1.49 999.99 998.5 4.97    40.38
##
##      se
## X1 0.64
```

```
describeBy(legosets$US_retailPrice, group = legosets$availability, mat = TRUE, skew = FALSE)
```

| ##      | item | group1                         | vars | n    | mean      | sd        | median | min   | max    | range | se         |
|---------|------|--------------------------------|------|------|-----------|-----------|--------|-------|--------|-------|------------|
| ## X11  | 1    | {Not specified}                | 1    | 2004 | 27.76015  | 38.92310  | 19.99  | 1.49  | 789.99 | 788.5 | 0.8694780  |
| ## X12  | 2    | Educational                    | 1    | 12   | 217.03333 | 108.17617 | 232.45 | 14.95 | 399.95 | 385.0 | 31.2277699 |
| ## X13  | 3    | Gift with Purchase at LEGO.com | 1    | 0    | NaN       | NA        | NA     | Inf   | -Inf   | -Inf  | NA         |
| ## X14  | 4    | Insiders Reward                | 1    | 0    | NaN       | NA        | NA     | Inf   | -Inf   | -Inf  | NA         |
| ## X15  | 5    | LEGO exclusive                 | 1    | 1268 | 65.12353  | 112.88247 | 14.99  | 1.99  | 999.99 | 998.0 | 3.1700555  |
| ## X16  | 6    | LEGOLAND exclusive             | 1    | 2    | 4.99000   | 0.00000   | 4.99   | 4.99  | 4.99   | 0.0   | 0.0000000  |
| ## X17  | 7    | Not sold                       | 1    | 1    | 12.99000  | NA        | 12.99  | 12.99 | 12.99  | 0.0   | NA         |
| ## X18  | 8    | Promotional                    | 1    | 5    | 4.79000   | 0.83666   | 4.99   | 3.99  | 5.99   | 2.0   | 0.3741657  |
| ## X19  | 9    | Promotional (Airline)          | 1    | 0    | NaN       | NA        | NA     | Inf   | -Inf   | -Inf  | NA         |
| ## X110 | 10   | Retail                         | 1    | 5062 | 40.59401  | 40.96294  | 29.99  | 1.99  | 699.99 | 698.0 | 0.5757448  |
| ## X111 | 11   | Retail - limited               | 1    | 319  | 63.40755  | 69.70365  | 39.99  | 2.49  | 449.99 | 447.5 | 3.9026552  |



# Grammar of Graphics



# Data Visualizations with ggplot2



- `ggplot2` is an R package that provides an alternative framework based upon Wilkinson's (2005) Grammar of Graphics.
- `ggplot2` is, in general, more flexible for creating "prettier" and complex plots.
- Works by creating layers of different types of objects/geometries (i.e. bars, points, lines, polygons, etc.) `ggplot2` has at least three ways of creating plots:
  1. `qplot`
  2. `ggplot(...) + geom_XXX(...) + ...`
  3. `ggplot(...) + layer(...)`
- We will focus only on the second.



# Parts of a `ggplot2` Statement



- Data

```
ggplot(myDataFrame, aes(x=x, y=y))
```

- Layers

```
geom_point(), geom_histogram()
```

- Facets

```
facet_wrap(~ cut), facet_grid(~ cut)
```

- Scales

```
scale_y_log10()
```

- Other options

```
ggtitle('my title'), ylim(c(0, 10000)), xlab('x-axis label')
```



# Lots of geoms



```
ls('package:ggplot2')[grep('^geom_', ls('package:ggplot2'))]
```

```
## [1] "geom_abline"      "geom_area"        "geom_bar"         "geom_bin_2d"
## [5] "geom_bin2d"       "geom_blank"       "geom_boxplot"     "geom_col"
## [9] "geom_contour"     "geom_contour_filled" "geom_count"      "geom_crossbar"
## [13] "geom_curve"       "geom_density"     "geom_density_2d"  "geom_density_2d_filled"
## [17] "geom_density2d"   "geom_density2d_filled" "geom_dotplot"    "geom_errorbar"
## [21] "geom_errorbarh"   "geom_freqpoly"    "geom_function"    "geom_hex"
## [25] "geom_histogram"   "geom_hline"       "geom_jitter"      "geom_label"
## [29] "geom_line"        "geom_linerange"   "geom_map"         "geom_path"
## [33] "geom_point"       "geom_pointrange"  "geom_polygon"     "geom_qq"
## [37] "geom_qq_line"     "geom_quantile"    "geom_raster"      "geom_rect"
## [41] "geom_ribbon"      "geom_rug"         "geom_segment"     "geom_sf"
## [45] "geom_sf_label"    "geom_sf_text"     "geom_smooth"      "geom_spoke"
## [49] "geom_step"        "geom_text"        "geom_tile"        "geom_violin"
## [53] "geom_vline"
```



# Data Visualization Cheat Sheet



## Data Visualization with ggplot2 : : CHEAT SHEET

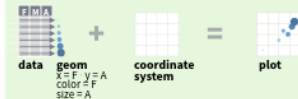


### Basics

ggplot2 is based on the **grammar of graphics**, the idea that you can build every graph from the same components: a **data set**, a **coordinate system**, and **geoms**—visual marks that represent data points.



To display values, map variables in the data to visual properties of the geom (**aesthetics**) like **size**, **color**, and **x** and **y** locations.



Complete the template below to build a graph.

```
ggplot(data = <DATA>) +  
  <GEOM_FUNCTION>(mapping = aes(<MAPPINGS>)) +  
  stat = <STAT>, position = <POSITION> +  
  <COORDINATE_FUNCTION> +  
  <FACET_FUNCTION> +  
  <SCALE_FUNCTION> +  
  <THEME_FUNCTION>
```

ggplot(data = mpg, aes(x = cty, y = hwy)) Begins a plot that you finish by adding layers to. Add one geom function per layer.

ggplot(x = cty, y = hwy, data = mpg, geom = "point") Creates a complete plot with given data, geom, and mappings. Supplies many useful defaults.

last\_plot() Returns the last plot

ggsave("plot.png", width = 5, height = 5) Saves last plot as 5' x 5' file named "plot.png" in working directory. Matches file type to file extension.

### Geoms

Use a geom function to represent data points, use the geom's aesthetic properties to represent variables. Each function returns a layer.

#### GRAPHICAL PRIMITIVES

```
a <- ggplot(economics, aes(date, unemployment))  
b <- ggplot(seals, aes(x = long, y = lat))
```

**a + geom\_blank()**  
(Useful for expanding limits)

**b + geom\_curve()**(aes(yend = lat + 1, xend = long + 1, curvature = z)) - x, xend, y, yend, alpha, angle, color, curvature, linetype, size

**a + geom\_path()**(lineend = "butt", linejoin = "round", linemitre = 1)  
x, y, alpha, color, group, linetype, size

**a + geom\_polygon()**(aes(group = group))  
x, y, alpha, color, fill, group, linetype, size

**b + geom\_rect()**(aes(xmin = long, ymin = lat, xmax = long + 1, ymax = lat + 1)) - xmax, xmin, ymax, ymin, alpha, color, fill, linetype, size

**a + geom\_ribbon()**(aes(ymin = unemployment - 900, ymax = unemployment + 900)) - x, ymax, ymin, alpha, color, fill, group, linetype, size

#### LINE SEGMENTS

common aesthetics: x, y, alpha, color, linetype, size

```
b + geom_abline(aes(intercept = 0, slope = 1))  
b + geom_hline(aes(yintercept = lat))  
b + geom_vline(aes(xintercept = long))
```

```
b + geom_segment(aes(yend = lat + 1, xend = long + 1))  
b + geom_spoke(aes(angle = 1:1155, radius = 1))
```

#### ONE VARIABLE continuous

```
c <- ggplot(mpg, aes(hwy)); c2 <- ggplot(mpg)
```

**c + geom\_area()**(stat = "bin")  
x, y, alpha, color, fill, linetype, size

**c + geom\_density()**(kernel = "gaussian")  
x, y, alpha, color, fill, group, linetype, size, weight

**c + geom\_dotplot()**  
x, y, alpha, color, fill

**c + geom\_freqpoly()** x, y, alpha, color, group, linetype, size

**c + geom\_histogram()**(binwidth = 5) x, y, alpha, color, fill, linetype, size, weight

**c2 + geom\_qq()**(aes(sample = hwy)) x, y, alpha, color, fill, linetype, size, weight

#### discrete

```
d <- ggplot(mpg, aes(fit))
```

**d + geom\_bar()**  
x, alpha, color, fill, linetype, size, weight

#### TWO VARIABLES

**continuous x, continuous y**  
e <- ggplot(mpg, aes(cty, hwy))

**e + geom\_label()**(aes(label = cty, nudge\_x = 1, nudge\_y = 1, check\_overlap = TRUE)) x, y, label, alpha, angle, color, family, fontface, hjust, lineheight, size, vjust

**e + geom\_jitter()**(height = 2, width = 2)  
x, y, alpha, color, fill, shape, size

**e + geom\_point()**, x, y, alpha, color, fill, shape, size, stroke

**e + geom\_quantile()**, x, y, alpha, color, group, linetype, size, weight

**e + geom\_rug()**(sides = "bl"), x, y, alpha, color, linetype, size

**e + geom\_smooth()**(method = lm), x, y, alpha, color, fill, group, linetype, size, weight

**e + geom\_text()**(aes(label = cty, nudge\_x = 1, nudge\_y = 1, check\_overlap = TRUE)) x, y, label, alpha, angle, color, family, fontface, hjust, lineheight, size, vjust

#### discrete x, continuous y

```
f <- ggplot(mpg, aes(class, hwy))
```

**f + geom\_col()**, x, y, alpha, color, fill, group, linetype, size

**f + geom\_boxplot()**, x, y, lower, middle, upper, ymax, ymin, alpha, color, fill, group, linetype, shape, size, weight

**f + geom\_dotplot()**(binaxis = "y", stackdir = "center"), x, y, alpha, color, fill, group

**f + geom\_violin()**(scale = "area"), x, y, alpha, color, fill, group, linetype, size, weight

#### discrete x, discrete y

```
g <- ggplot(diamonds, aes(cut, color))
```

**g + geom\_count()**, x, y, alpha, color, fill, shape, size, stroke

#### THREE VARIABLES

```
seals$z <- with(seals, sqrt(delta_long^2 + delta_lat^2)) l <- ggplot(seals, aes(long, lat))
```

**l + geom\_contour()**(aes(z = z))  
x, y, z, alpha, colour, group, linetype, size, weight

**l + geom\_tile()**(aes(fill = z)), x, y, alpha, color, fill, linetype, size, width

#### continuous bivariate distribution

```
h <- ggplot(diamonds, aes(carat, price))
```

**h + geom\_bin2d()**(binwidth = c(0.25, 500))  
x, y, alpha, color, fill, linetype, size, weight

**h + geom\_density2d()**  
x, y, alpha, colour, group, linetype, size

**h + geom\_hex()**  
x, y, alpha, colour, fill, size

#### continuous function

```
i <- ggplot(economics, aes(date, unemployment))
```

**i + geom\_area()**  
x, y, alpha, color, fill, linetype, size

**i + geom\_line()**  
x, y, alpha, color, group, linetype, size

**i + geom\_step()**(direction = "hv")  
x, y, alpha, color, group, linetype, size

#### visualizing error

```
df <- data.frame(grp = c("A", "B"), fit = 4.5, se = 1.2)  
j <- ggplot(df, aes(grp, fit, ymin = fit-se, ymax = fit+se))
```

**j + geom\_crossbar()**(fatten = 2)  
x, y, ymax, ymin, alpha, color, fill, group, linetype, size

**j + geom\_errorbar()**, x, ymax, ymin, alpha, color, group, linetype, size, width (also **geom\_errorbarh()**)

**j + geom\_linerange()**  
x, ymin, ymax, alpha, color, group, linetype, size

**j + geom\_pointrange()**  
x, y, ymin, ymax, alpha, color, fill, group, linetype, shape, size

#### maps

```
data <- data.frame(murder = USArrests$Murder,  
state = tolower(rownames(USArrests)))  
map <- map_data("state")  
k <- ggplot(data, aes(fill = murder))
```

**k + geom\_map()**(aes(map\_id = state), map = map)  
**+ expand\_limits()**(x = map\$long, y = map\$lat), map\_id, alpha, color, fill, linetype, size



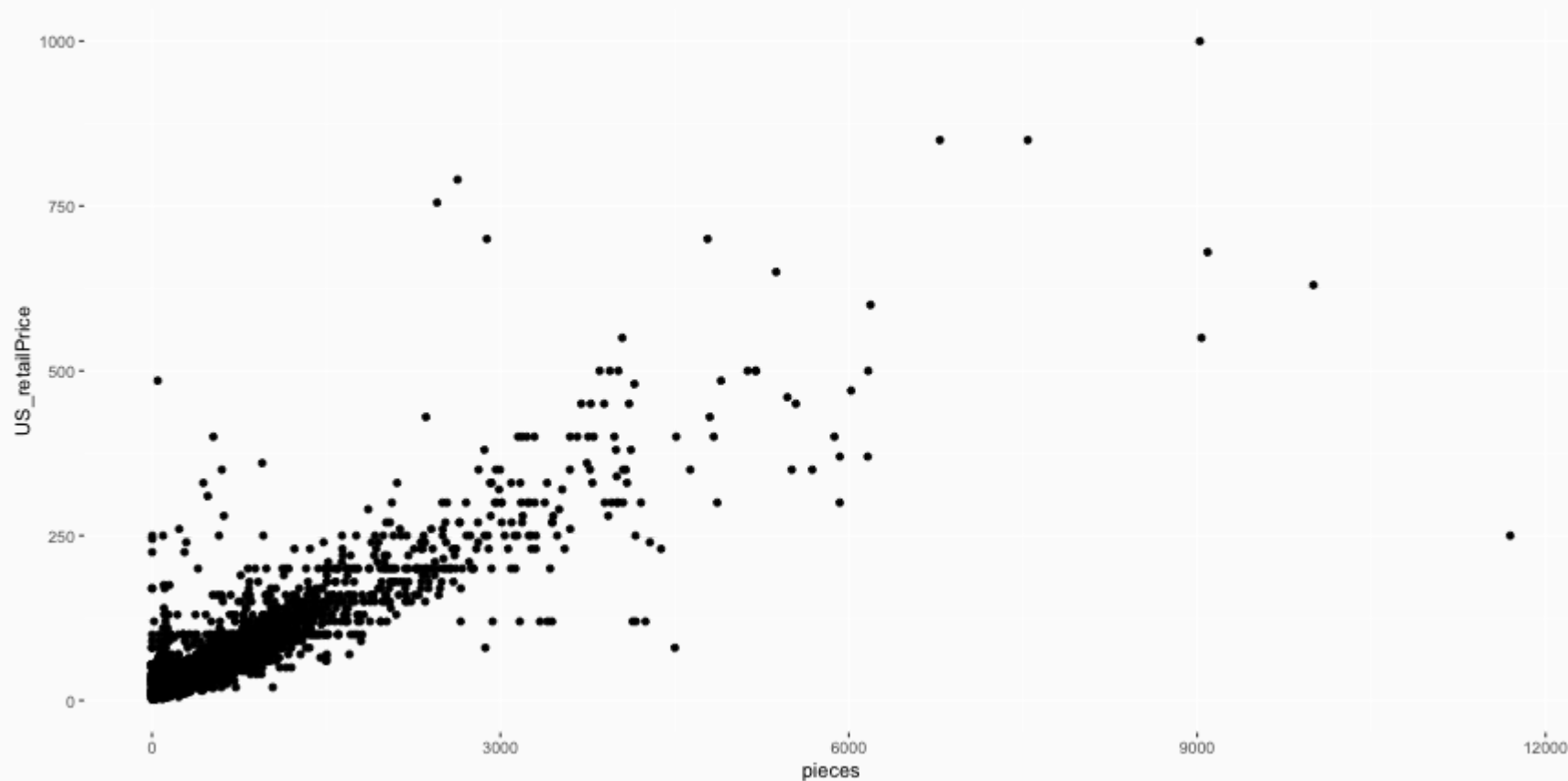
RStudio® is a trademark of RStudio, Inc. • CC BY SA RStudio • info@rstudio.com • 844-448-1212 • rstudio.com • Learn more at <http://ggplot2.tidyverse.org> • ggplot2 2.1.0 • Updated: 2016-11



# Scatterplot



```
ggplot(legosets, aes(x=pieces, y=US_retailPrice)) + geom_point()
```



# Scatterplot (cont.)



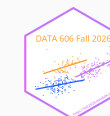
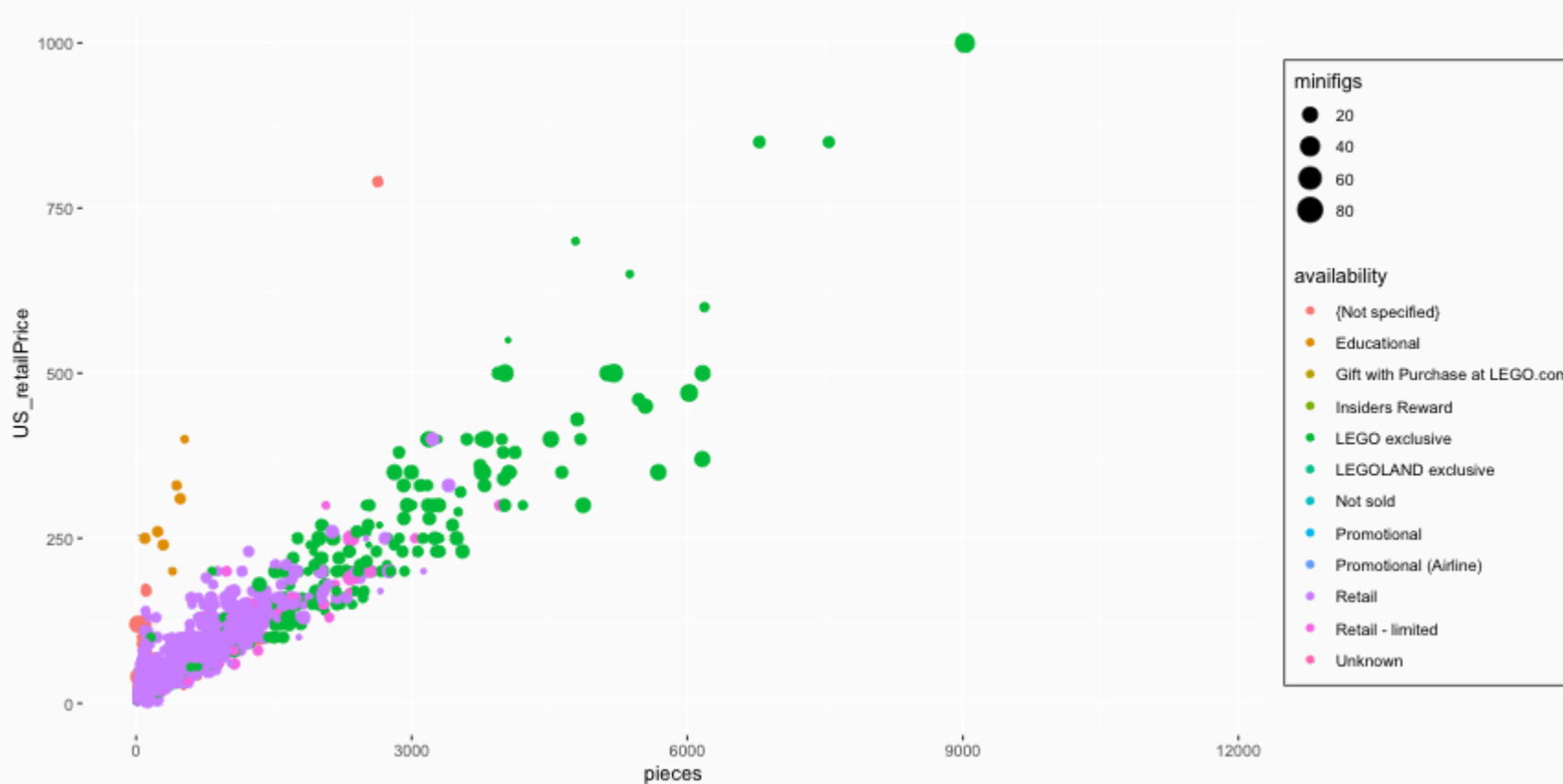
```
ggplot(legosets, aes(x=pieces, y=US_retailPrice, color=availability)) + geom_point()
```



# Scatterplot (cont.)



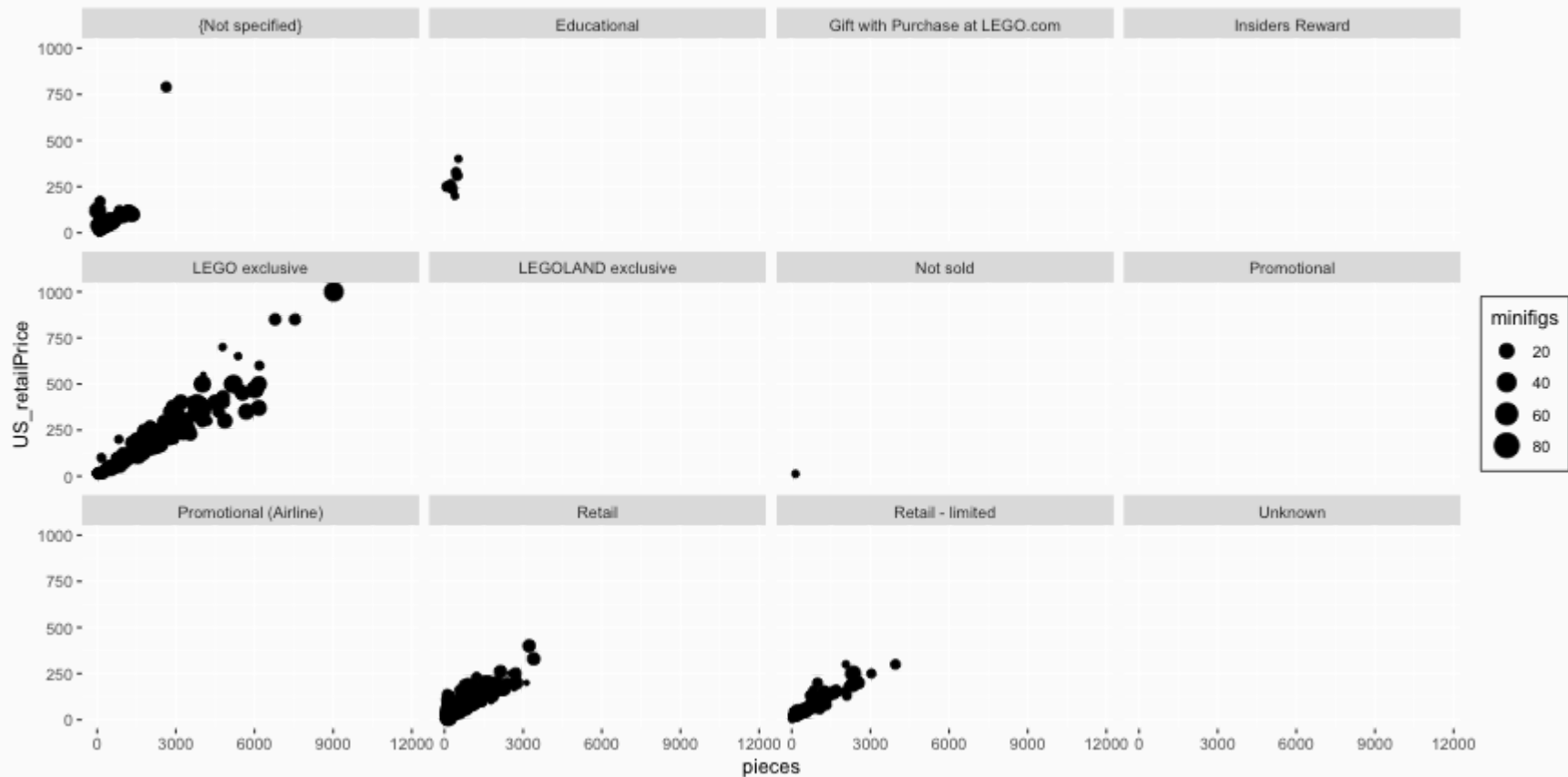
```
ggplot(legosets, aes(x=pieces, y=US_retailPrice, size=minifigs, color=availability)) + geom_point()
```



# Scatterplot (cont.)



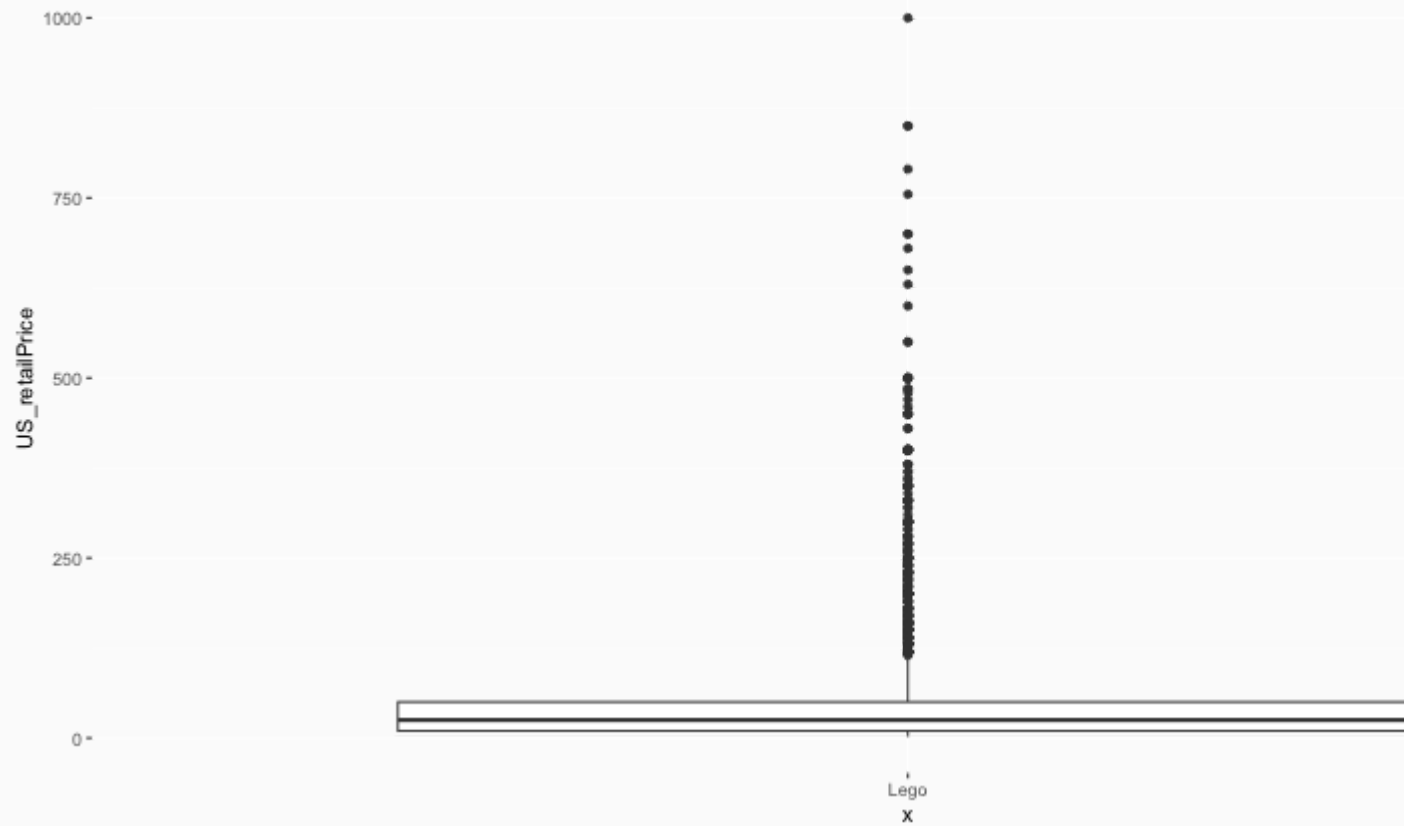
```
ggplot(legosets, aes(x=pieces, y=US_retailPrice, size=minifigs)) + geom_point() + facet_wrap(~ availability)
```



# Boxplots



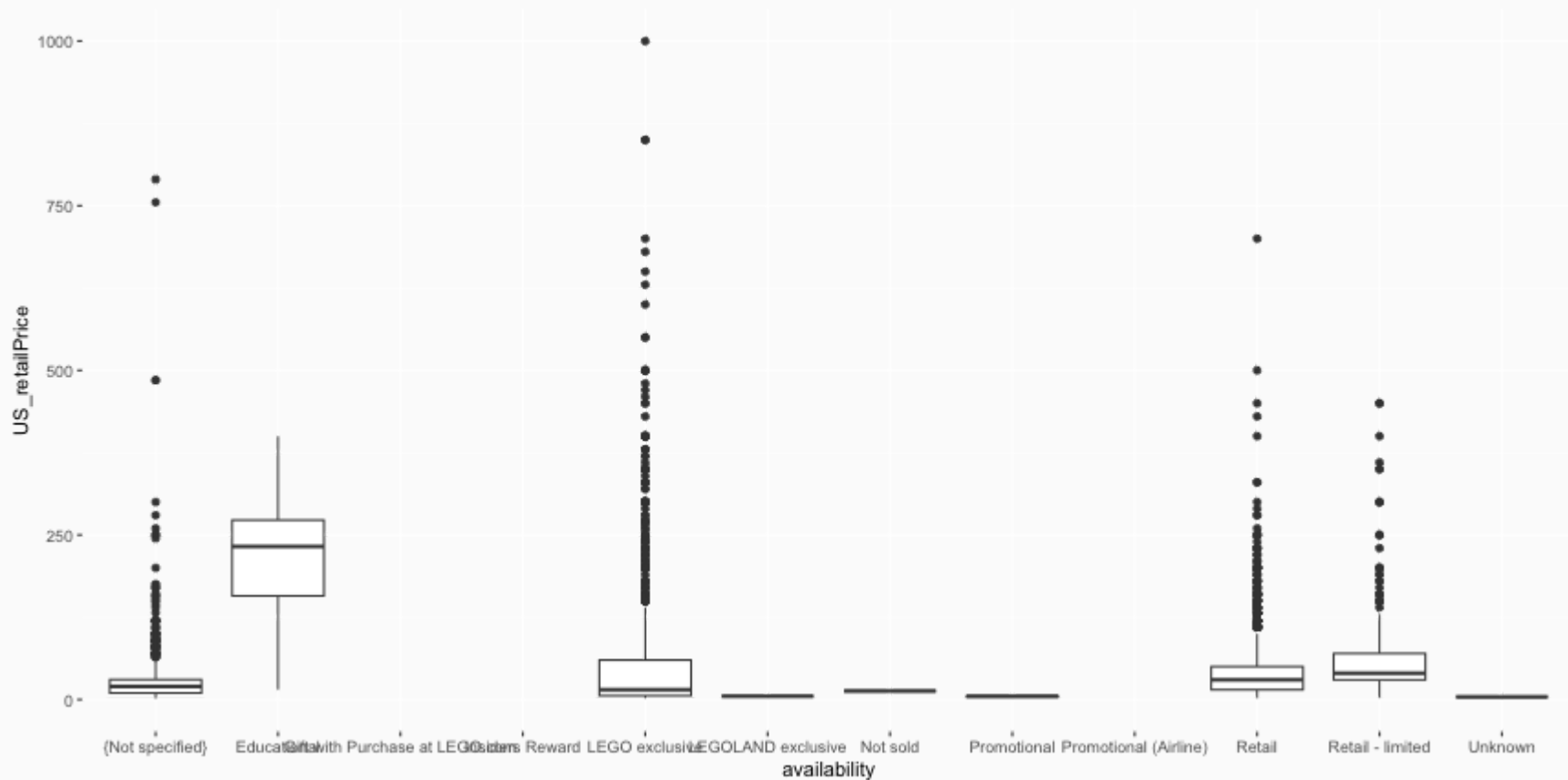
```
ggplot(legosets, aes(x='Lego', y=US_retailPrice)) + geom_boxplot()
```



# Boxplots (cont.)



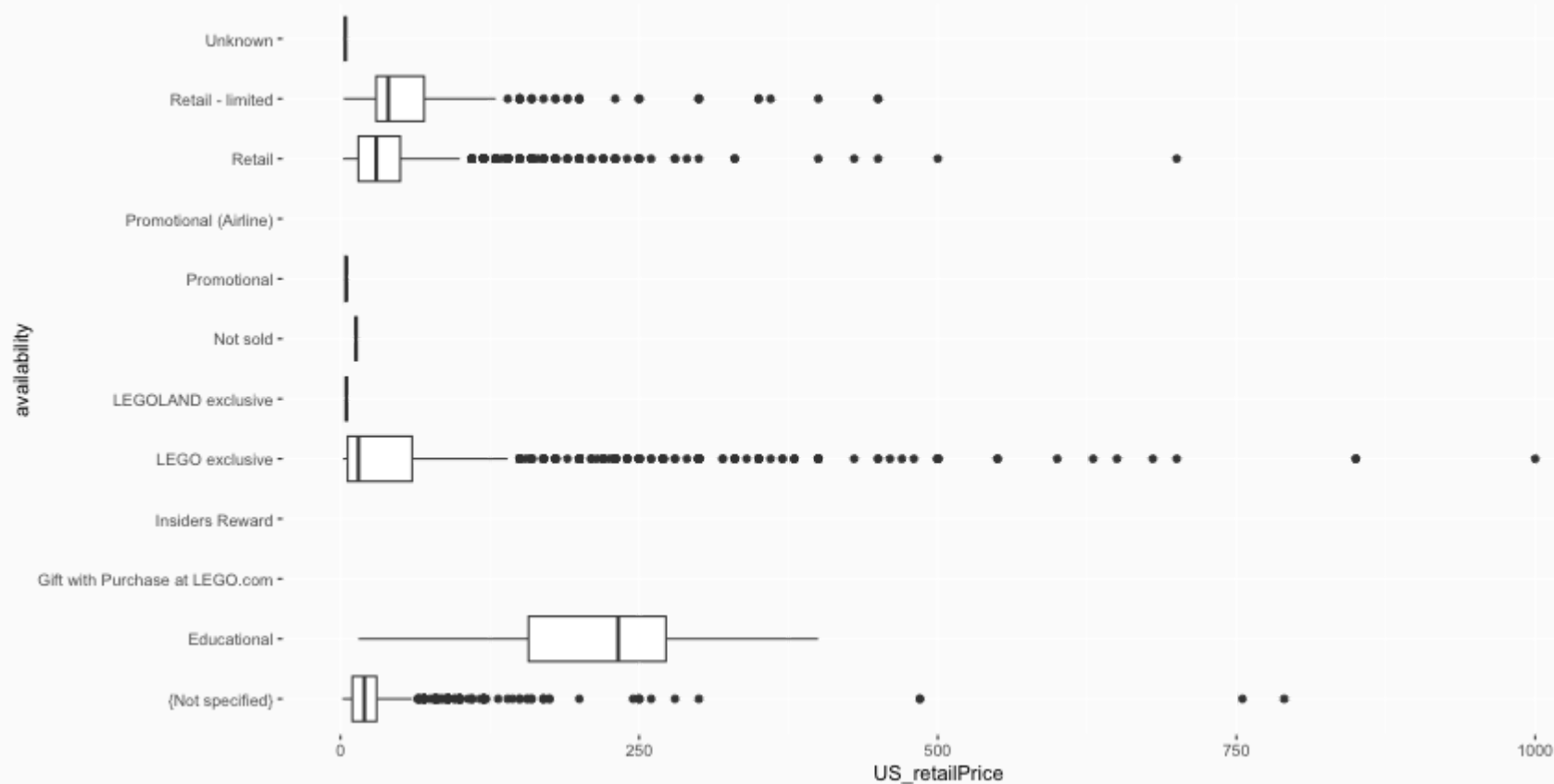
```
ggplot(legosets, aes(x=availability, y=US_retailPrice)) + geom_boxplot()
```



# Boxplot (cont.)



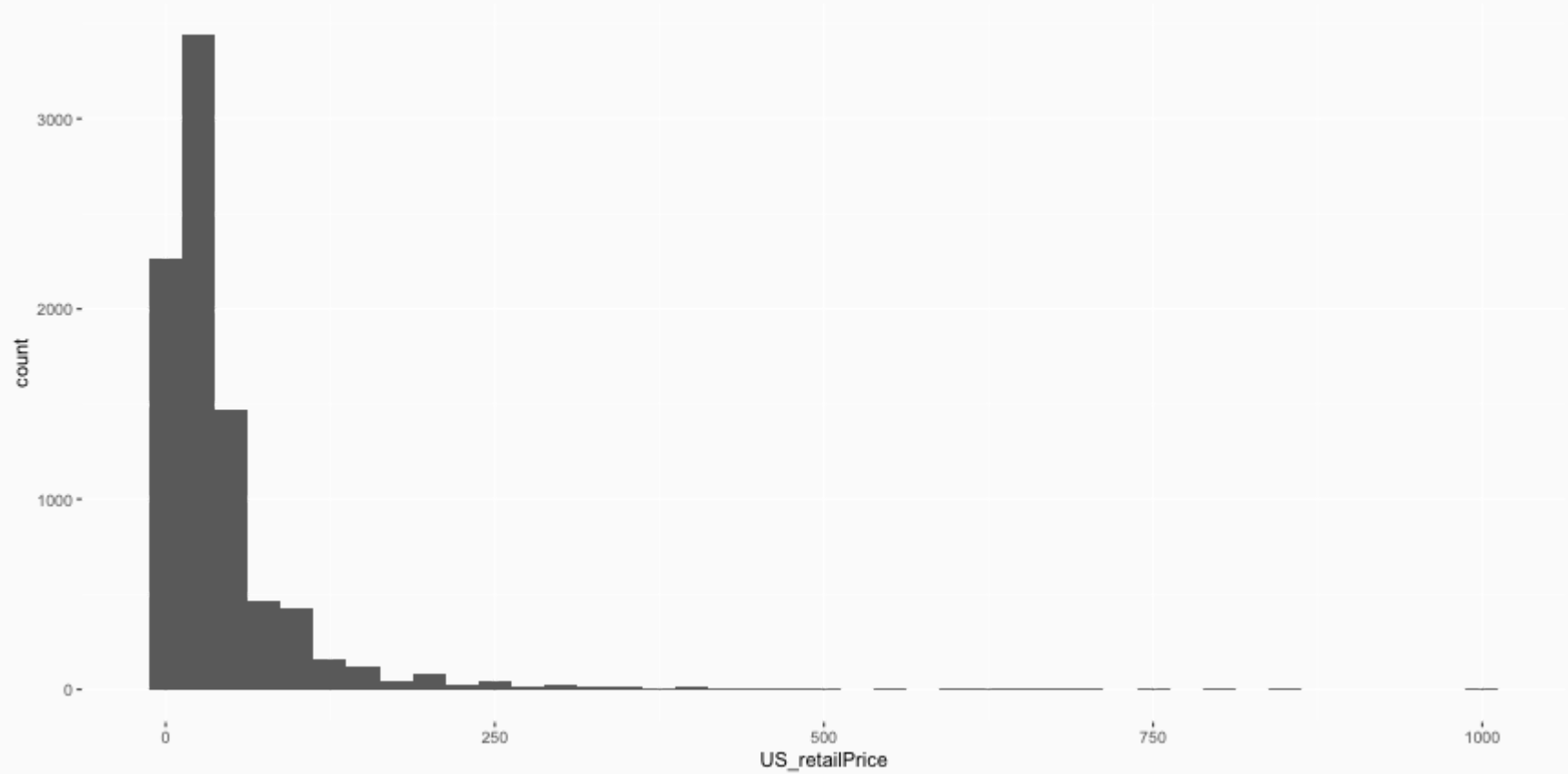
```
ggplot(legosets, aes(x=availability, y=US_retailPrice)) + geom_boxplot() + coord_flip()
```



# Histograms



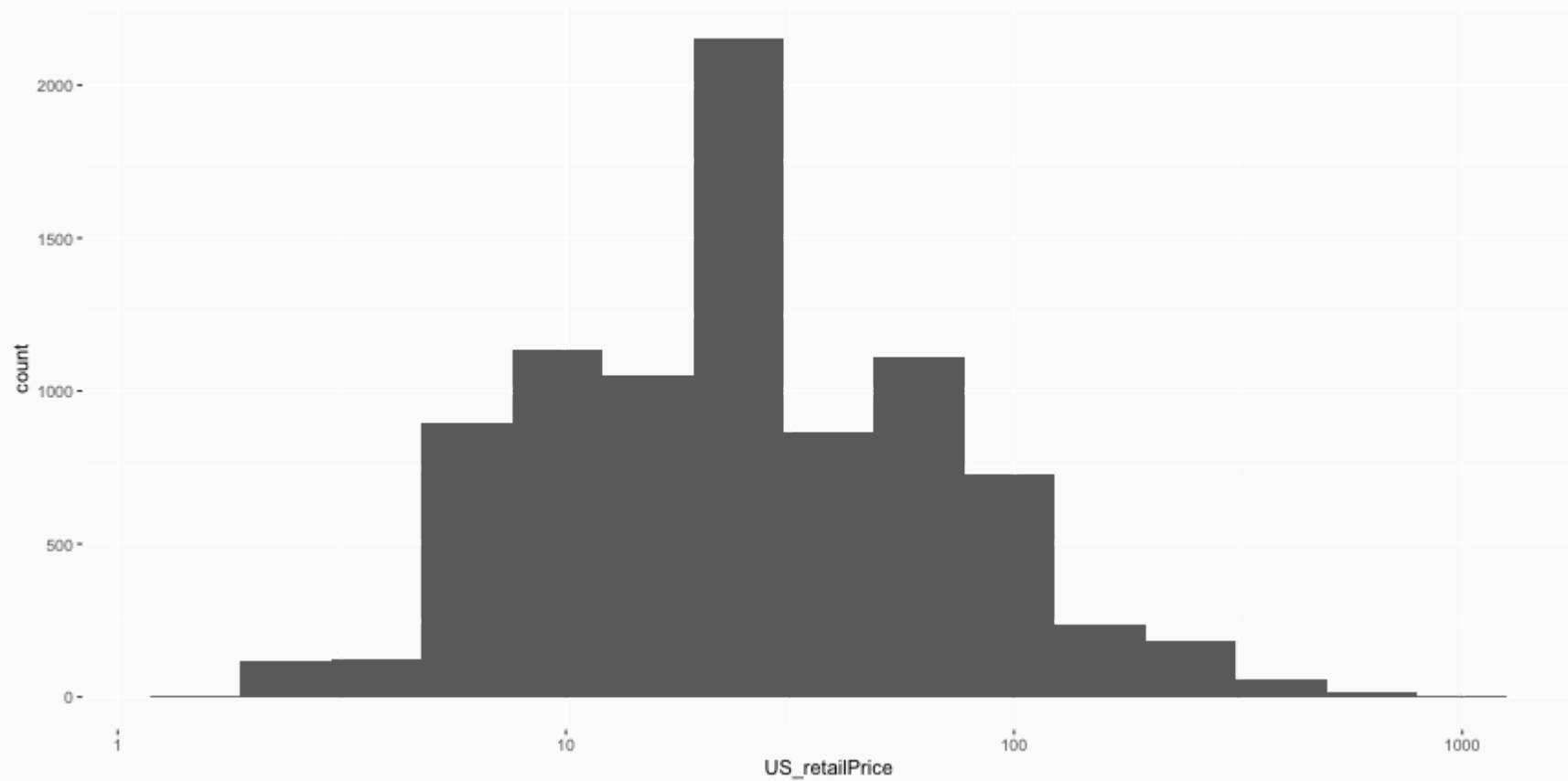
```
ggplot(legosets, aes(x = US_retailPrice)) + geom_histogram(binwidth = 25)
```



# Histograms (cont.)



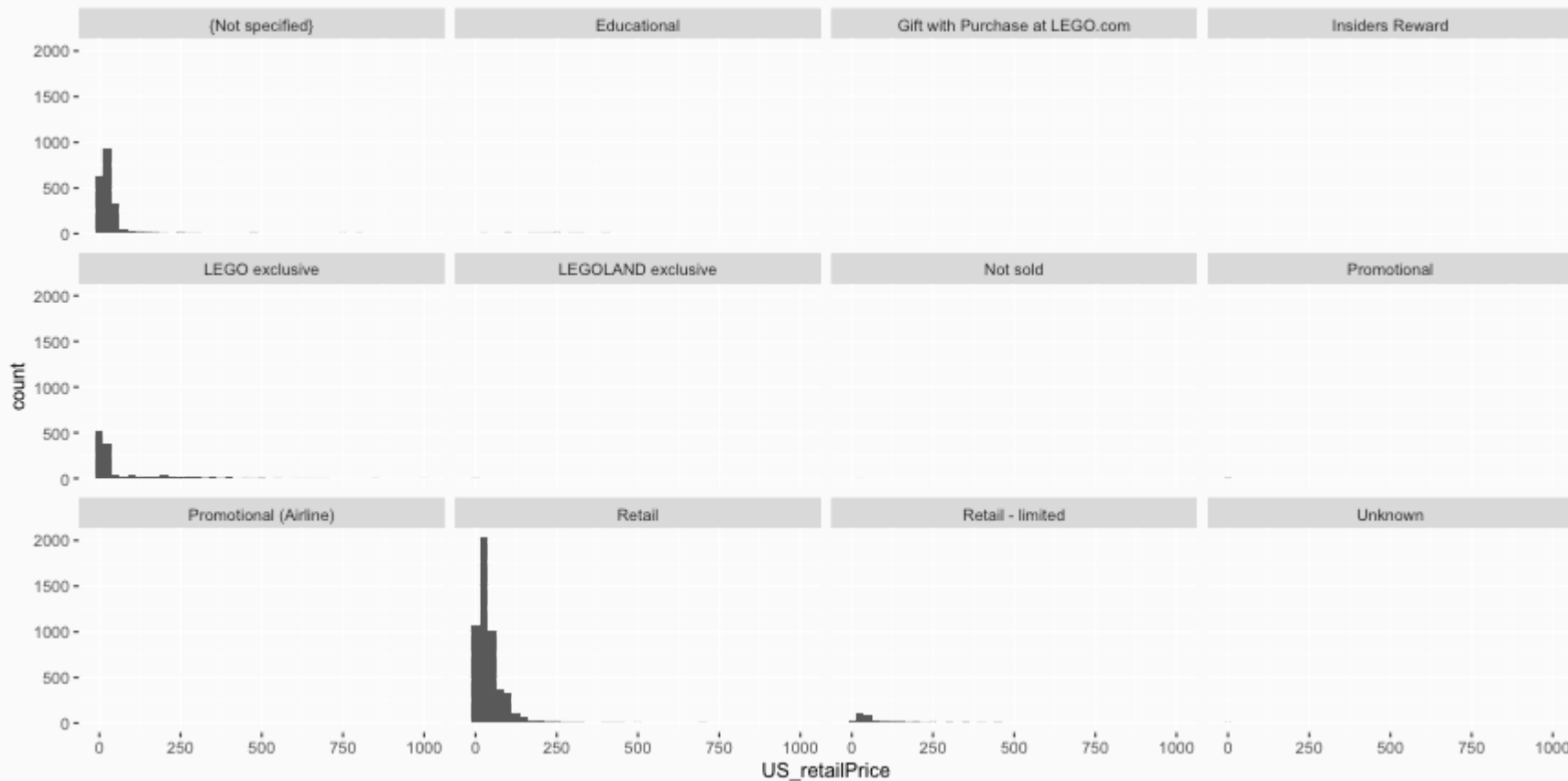
```
ggplot(legosets, aes(x = US_retailPrice)) + geom_histogram(bins = 15) + scale_x_log10()
```



# Histograms (cont.)



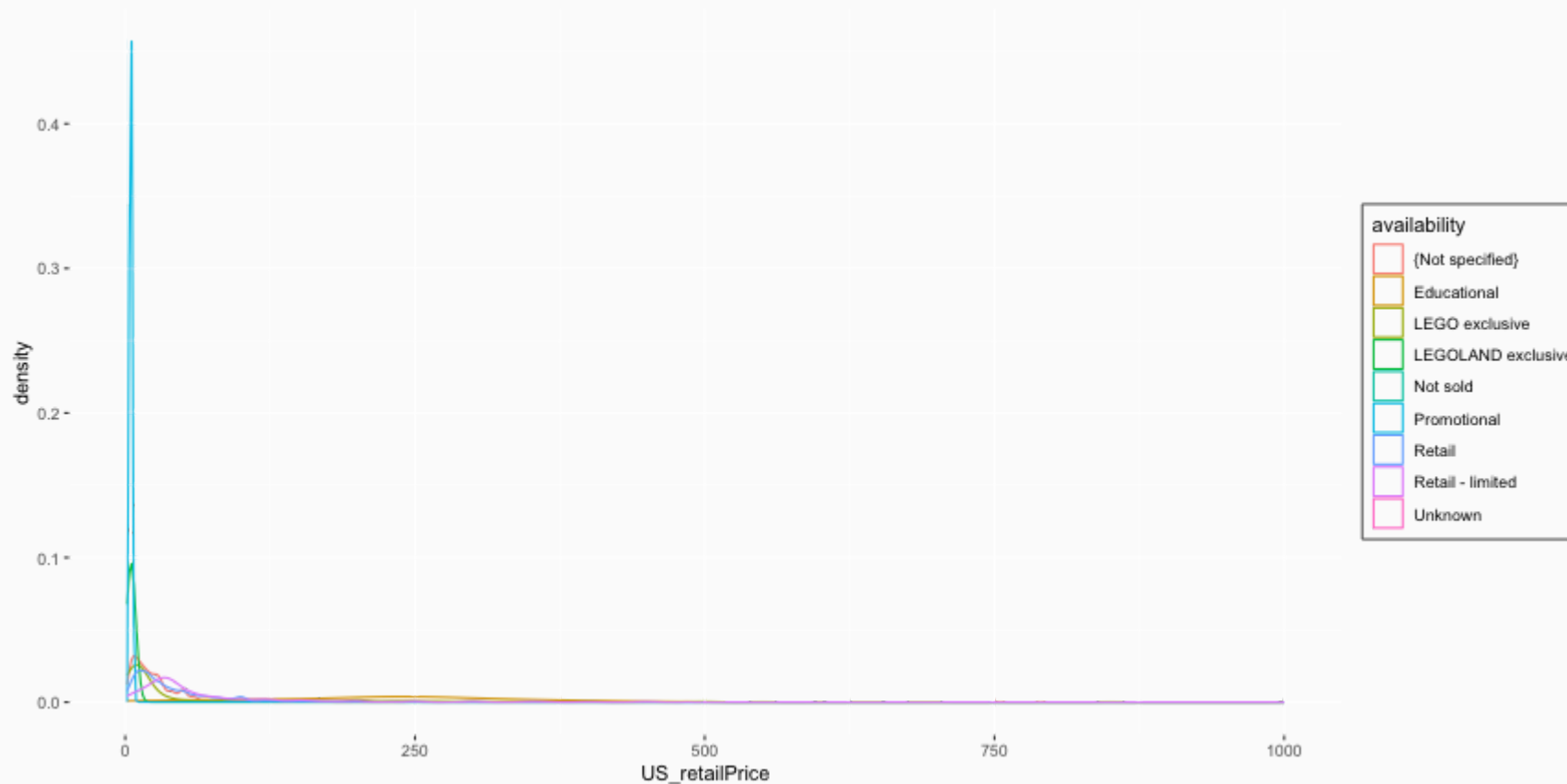
```
ggplot(legosets, aes(x = US_retailPrice)) + geom_histogram(binwidth = 25) + facet_wrap(~ availability)
```



# Density Plots



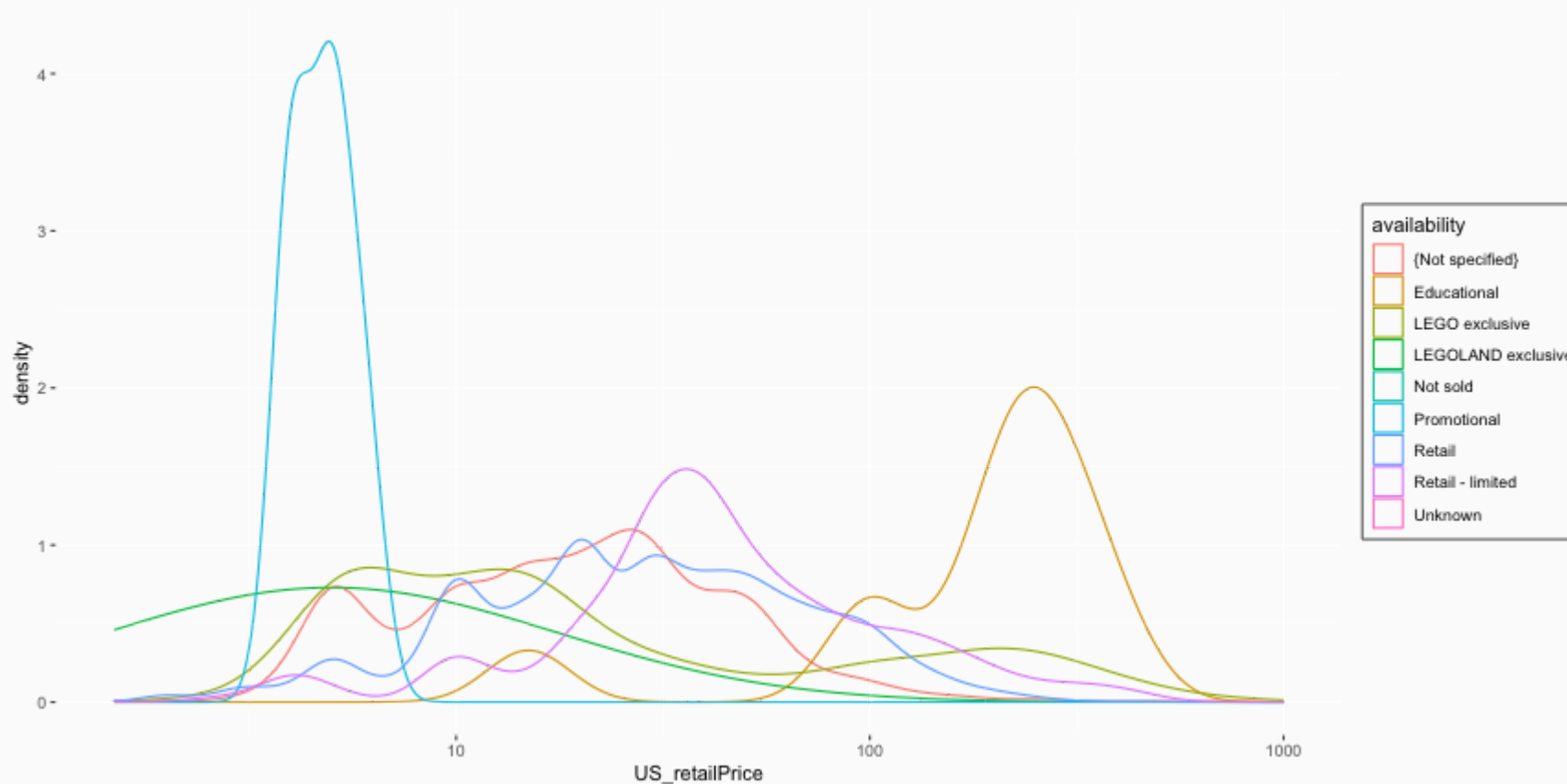
```
ggplot(legosets, aes(x = US_retailPrice, color = availability)) + geom_density()
```



# Density Plots (cont.)



```
ggplot(legosets, aes(x = US_retailPrice, color = availability)) + geom_density() + scale_x_log10()
```



idea and design: Christian Burkhart  
design advice: Ida Aarnio



Likert scales are a type of questionnaire where respondents are asked to rate items on scales usually ranging from four to seven levels (e.g. strongly disagree to strongly agree).

```
library(likert)
library(reshape)
data(pisaitems)
items24 <- pisaitems[,substr(names(pisaitems), 1,5) == 'ST24Q']
items24 <- rename(items24, c(
  ST24Q01="I read only if I have to.",
  ST24Q02="Reading is one of my favorite hobbies.",
  ST24Q03="I like talking about books with other people.",
  ST24Q04="I find it hard to finish books.",
  ST24Q05="I feel happy if I receive a book as a present.",
  ST24Q06="For me, reading is a waste of time.",
  ST24Q07="I enjoy going to a bookstore or a library.",
  ST24Q08="I read only to get information that I need.",
  ST24Q09="I cannot sit still and read for more than a few minutes.",
  ST24Q10="I like to express my opinions about books I have read.",
  ST24Q11="I like to exchange books with my friends."))
```





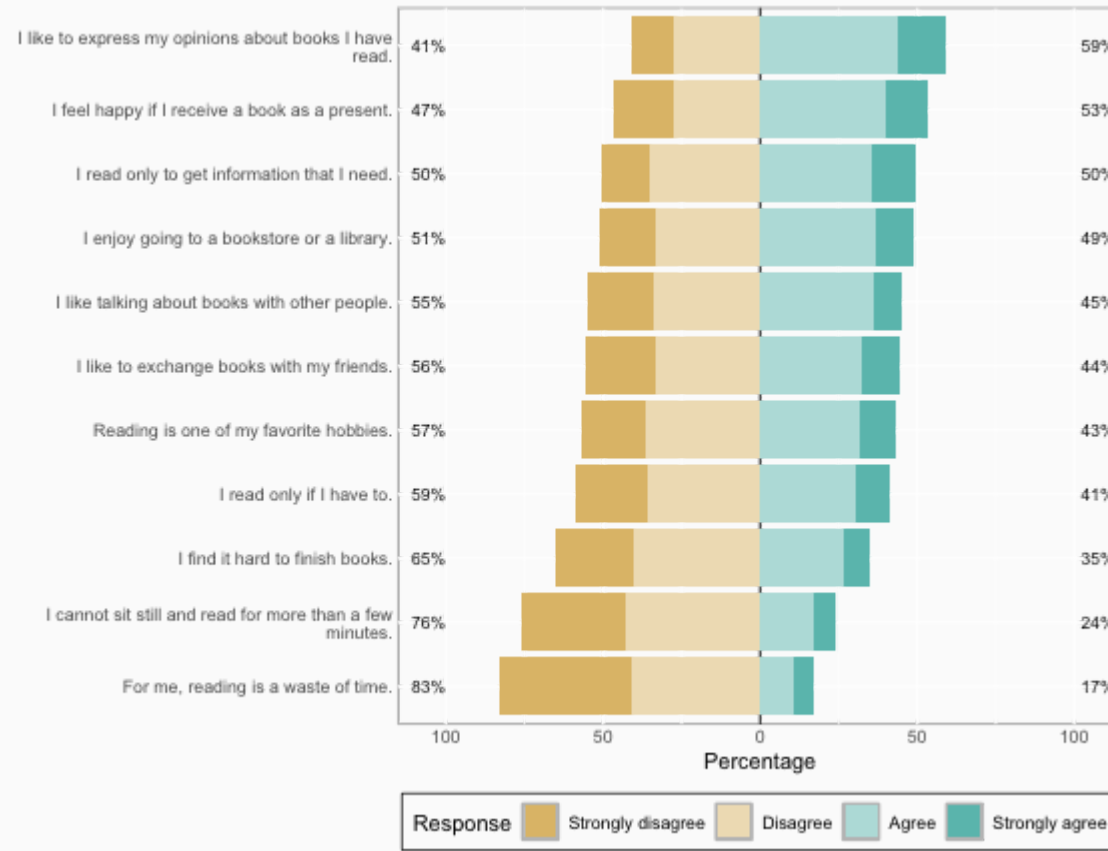
```
l24 <- likert(items24)
summary(l24)
```

| ##    | Item                                                     | low      | neutral | high     | mean     | sd        |
|-------|----------------------------------------------------------|----------|---------|----------|----------|-----------|
| ## 10 | I like to express my opinions about books I have read.   | 41.07516 | 0       | 58.92484 | 2.604913 | 0.9009968 |
| ## 5  | I feel happy if I receive a book as a present.           | 46.93475 | 0       | 53.06525 | 2.466751 | 0.9446590 |
| ## 8  | I read only to get information that I need.              | 50.39874 | 0       | 49.60126 | 2.484616 | 0.9089688 |
| ## 7  | I enjoy going to a bookstore or a library.               | 51.21231 | 0       | 48.78769 | 2.428508 | 0.9164136 |
| ## 3  | I like talking about books with other people.            | 54.99129 | 0       | 45.00871 | 2.328049 | 0.9090326 |
| ## 11 | I like to exchange books with my friends.                | 55.54115 | 0       | 44.45885 | 2.343193 | 0.9609234 |
| ## 2  | Reading is one of my favorite hobbies.                   | 56.64470 | 0       | 43.35530 | 2.344530 | 0.9277495 |
| ## 1  | I read only if I have to.                                | 58.72868 | 0       | 41.27132 | 2.291811 | 0.9369023 |
| ## 4  | I find it hard to finish books.                          | 65.35125 | 0       | 34.64875 | 2.178299 | 0.8991628 |
| ## 9  | I cannot sit still and read for more than a few minutes. | 76.24524 | 0       | 23.75476 | 1.974736 | 0.8793028 |
| ## 6  | For me, reading is a waste of time.                      | 82.88729 | 0       | 17.11271 | 1.810093 | 0.8611554 |

# likert Plots

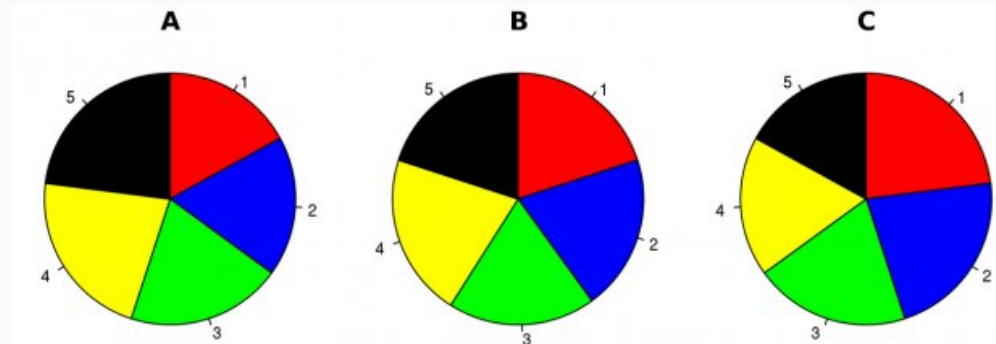


```
plot(l24)
```



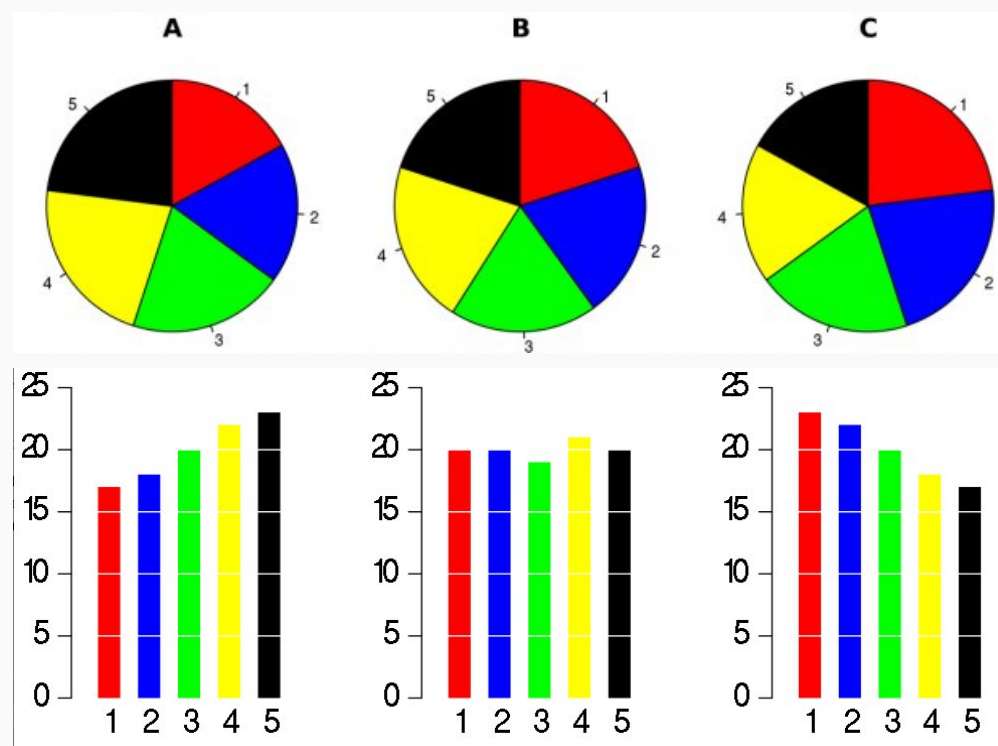
# Pie Charts

There is only one pie chart in *OpenIntro Statistics* (Diez, Barr, & Çetinkaya-Rundel, 2015, p. 48). Consider the following three pie charts that represent the preference of five different colors. Is there a difference between the three pie charts? This is probably a difficult to answer.



# Pie Charts

There is only one pie chart in *OpenIntro Statistics* (Diez, Barr, & Çetinkaya-Rundel, 2015, p. 48). Consider the following three pie charts that represent the preference of five different colors. Is there a difference between the three pie charts? This is probably a difficult to answer.



"There is no data that can be displayed in a pie chart that cannot better be displayed in some other type of chart"

John Tukey

# Additional Resources

For data wrangling:

- dplyr website: <https://dplyr.tidyverse.org>
- R for Data Science book: <https://r4ds.had.co.nz/wrangle-intro.html>
- Wrangling penguins tutorial: <https://allisonhorst.shinyapps.io/dplyr-learnr/#section-welcome>
- Data transformation cheat sheet: <https://github.com/rstudio/cheatsheets/raw/master/data-transformation.pdf>

For data visualization:

- ggplot2 website: <https://ggplot2.tidyverse.org>
- R for Data Science book: <https://r4ds.had.co.nz/data-visualisation.html>
- R Graphics Cookbook: <https://r-graphics.org>
- Data visualization cheat sheet: <https://github.com/rstudio/cheatsheets/raw/master/data-visualization-2.1.pdf>

# One Minute Paper

1. What was the most important thing you learned during this class?
2. What important question remains unanswered for you?



Scan the QR code or click here to  
complete the One Minute Paper

